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RESEARCH IN ELECTROMAGNETICS, PLASMAS AND NETWORKS

by

L. B. Felsen

Final Report No. PIBMRI-1112-63
For Item I of
Contract No. AF-19(604)-4143
Project No. 5635, Task No. 56350
for
Electronics Research Directorate
Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
L. G. Hanscom Field
Bedford, Massachusetts

February 13, 1963

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FINAL REPORT
- ITEM I -

RESEARCH IN ELECTROMAGNETICS, PLASMAS AND NETWORKS

by

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Title Page
Acknowledgement
Table of Contents
54 Pages of Text

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Prepared for
Electronics Research Directorate
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Table of Contents

Table of Contents (page 2)

A. Preface

This Final Report summarizes the work performed during the period from July 1, 1958 to December 31, 1962, under the Electromagnetics and Networks phase (Item I) of Contract No. AF-19(604)-4143. The other phases have already been terminated and are described in an earlier Final Report.*

The scope of the research activities, theoretical in nature, can be assessed from the Table of Contents. Summary descriptions are given in the text and reference is made to the more detailed treatments which have been reported elsewhere. Certain research phases which have not been completed will be continued under the successor Contract No. AF-19(628)-2357. Two separate programs may be distinguished: electromagnetics and network theory. The electromagnetics program has been concerned with a broad study of wave propagation and diffraction phenomena in layered or inhomogeneous media; in anisotropic media; and by variously shaped objects having constant or variable surface impedance properties. Many of these analyses have been motivated by current topics involving plasmas, and such practical applications as radio communication with, or radar detection of, plasma sheathed space vehicles, plasma diagnostics, etc., have been kept in mind when devising idealized prototype problems which can be subjected to rigorous analysis. A variety of novel wave phenomena have been discovered during these investigations and are described at the appropriate places in the text.

The networks program summarized in this report covers many different areas. While some of the results are extensions of already well-established classical theorems, others contain answers to original and previously unsolved problems. In particular, a firm structure for Darlington synthesis has finally been erected. Also, the outstanding problem of optimum multiple-channel filter design has been solved completely for the case of rational spectral densities. This latter result supersedes the work in Wiener's book and in most of the periodicals devoted to smoothing and prediction. Most topics are discussed briefly with emphasis mainly on their pertinent contributions.

The principal contributors to Phase I of this Contract have been:

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* N. Marcuvitz, "Theoretical and Exploratory Experimental Research in Electromagnetics, Networks and Related Solid State, and Plasma Topics", Microw. Res. Inst., Polytech. Inst. of Brooklyn, Final Report PIBMRI-806-60, Sept. 1960.

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Their names appear under the headings of the sections which they have contributed.

B. Radiation and Diffraction Problems in Anisotropic Regions

1. Introduction

While problems of electromagnetic plane wave propagation in anisotropic media have been explored in some detail in connection with light propagation through crystals, radio wave transmission in the presence of the ionosphere (gyroelectric case) or microwave propagation through ferrites (gyromagnetic case), much less work has been done concerning the radiation from localized sources in or near such media. A principal reason for this paucity of results (even at the time of initiation of this Contract) is the complication of the radiation and diffraction processes encountered when sources or obstacles are embedded in anisotropic regions characterized by a tensor dielectric constant and (or) permeability. Little incentive was present for engaging in this difficult analysis since applications were generally concerned with sources located exterior to, and removed "far" from, the anisotropic domain. The source field could therefore be represented asymptotically by its local plane wave constituents, and the latter were then traced through the region of anisotropy (ray-tracing). More recent applications, however, require the knowledge of the radiation characteristics from sources located inside the medium or near its boundaries, and of the effect of perturbing objects on the radiation or diffraction field. For example, these results are of interest for radio wave transmission to and from satellites passing through the ionosphere; radiation from plasma-sheathed space vehicles subjected to an externally impressed magnetic field to eliminate communication blackout; excitation of whistlers; radiation from charges moving in or near anisotropic plasmas or ferrites (Cerenkov problem); plasma diagnostics. With these and other applications in mind, a basic research program on radiation and diffraction in anisotropic regions was initiated. To render the mathematical analysis tractable, the anisotropic medium was assumed to be representable by a dielectric and (or) permeability tensor; conditions under which actual plasmas or ferrites can be so represented are well-known in the literature[1,2] and need not be stated here.

The simplest type of anisotropy is the uniaxial; in an orthogonal coordinate system, with one of the coordinates parallel to the "optic axis" (z-axis) in the medium, the dielectric tensor $\underset{\approx}{\varepsilon}$ and (or) the permeability tensor $\underset{\approx}{\mu}$ is diagonal, with two identical elements along (x, y) and a different element along z. In vector notation:

$$\underset{\approx}{\varepsilon} = \varepsilon_1 \underset{\approx}{1}_t + \varepsilon_2 \underline{z}_o \underline{z}_o \quad , \qquad \underset{\approx}{\mu} = \mu_1 \underset{\approx}{1}_t + \mu_2 \underline{z}_o \underline{z}_o \quad , \qquad (1)$$

where $\underset{\approx}{1}_t$ is a unit dyadic in the domain transverse to z, and $\underline{z}_o$ is a unit vector in the

positive z-direction. In a plasma or ferrite subjected to a steady magnetic field $\underline{H}_o = \underline{z}_o H_o$, the uniaxial case obtains in the limit of infinite H_o, when the electrons are constrained to move parallel to the z-axis only. If the region is plane stratified along z, and transversely either unbounded, or bounded by a perfectly conducting cylindrical surface (of arbitrary cross section) whose axis is parallel to z, it is not difficult to show that the fields radiated by arbitrary source distributions can be decomposed into conventional E and H modes relative to the z-axis; the modal amplitudes satisfy transmission line equations which are only slightly different from those familiar from the theory of isotropic waveguides.[3] Radiation and diffraction problems in such regions can therefore be analyzed and formally solved by slight modifications of conventional methods; the asymptotic investigation of the radiation field, however, reveals phenomena which differ drastically from those encountered in isotropic media. Especially interesting is the case $\epsilon_2 < 0$ or $\mu_2 < 0$ which gives rise to focusing and shadow effects that are completely absent in isotropic regions (Sec. 2b). If the optic axis is perpendicular to the obstacle or boundary surface, certain two-dimensional diffraction problems can still be solved by relating them to equivalent isotropic problems (Sec. 2e). No such simple methods can be employed, however, when the optic axis is inclined arbitrarily with respect to the boundary.

These studies of radiation and diffraction in uniaxially anisotropic regions lend insight into some of the effects to be encountered under more general conditions provided that the results are phrased in a manner which highlights the physical mechanism of the process involved. At distances several wavelengths from the source, the rigorous expressions for the fields can be approximated asymptotically in terms of local plane wave contributions modified to account for the anisotropy. A most significant modification is the distinction between the direction of the phase propagation vector (phase gradient vector, $\underline{p} = \kappa \underline{z}_o + \sigma \underline{\rho}_o$) and the direction of the mean energy flow vector (Poynting or ray vector, $\underline{S} = \underline{\rho}_o S_\rho + \underline{z}_o S_z$) in a propagating plane wave. ($\underline{\rho}_o$ is the transverse vector coordinate). Their relation can be inferred from the refractive index diagram $n(\theta)$ vs. θ which gives the value of the refractive index n for different observation angles θ with respect to the optic axis: the endpoints of the vector $\underline{p}$ describe the $n(\theta)$ vs. θ surface, and the vector $\underline{S}$ is perpendicular to the surface.[4] In an isotropic medium ($\epsilon_1 = \epsilon_2$, $\mu_1 = \mu_2$), the refractive index surface is a sphere whence $\underline{p}$ and $\underline{S}$ are parallel. In a lossless homogeneous medium with a uniaxial dielectric tensor $\epsilon_1 \neq \epsilon_2$ appropriate to a plasma under the influence of an infinite external magnetic field, ϵ_1 is positive and ϵ_2 may be positive or negative. In the former case, for E modes along z, the n vs. θ surface is an ellipsoid while in the latter case,

it is a hyperboloid (Fig. 1).*

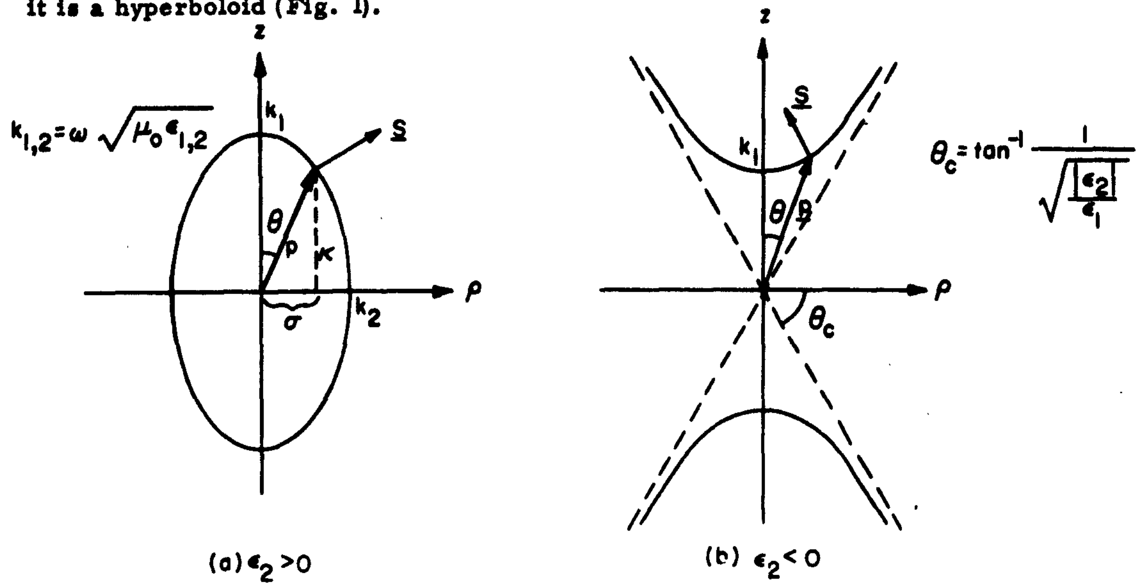


Fig. 1 - Refractive index curves for uniaxial plasma medium

$$\left(\epsilon_1 = \epsilon_o, \quad \epsilon_2 = \epsilon_o\left(1 - \frac{\omega_p^2}{\omega^2}\right)\right).$$

p and S are no longer parallel, and when $\epsilon_2 < 0$, the range of propagating rays is restricted to the conical region $\theta < \theta_c$; the region $\pi/2 > \theta > \theta_c$ is then a shadow region which has no counterpart in isotropic problems. (These remarks, made for the half-space $z > 0$, apply also to $z < 0$ since the refractive index surface is symmetrical about the $z = 0$ plane).

The refractive index curves provide a pictorial representation of the propagation characteristics of the anisotropic medium which is expressed analytically by the dispersion relation

$$H(\kappa, \sigma) = 0 \quad , \tag{2}$$

where κ and σ are the longitudinal and transverse components, respectively, of the phase propagation vector p. For the uniaxial medium, (2) is simply $\kappa^2 + (\sigma^2/\epsilon) = k_1^2$, $\epsilon = \epsilon_2/\epsilon_1$, whence one obtains the simple surfaces in Fig. 1. For a gyrotropic plasma (subjected to a finite external magnetic field H_o), where the dielectric tensor has the form

$$\underset{\sim}{\epsilon} \rightarrow \begin{pmatrix} \epsilon_1 & -i\epsilon_3 & 0 \\ i\epsilon_3 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_2 \end{pmatrix} \quad , \tag{3}$$

*H modes along z are not affected by the anisotropy, and the H mode refractive index surface is a sphere.

the dispersion relation (2) is much more complicated than for the uniaxial case, and the refractive index curves are separated into some eight categories[5] distinguished by the magnitude of the ratios ω/ω_p and ω/ω_c, where ω, ω_p and ω_c are the applied, plasma, and cyclotron frequencies, respectively (in a simplest description of a gyrotropic plasma, only the electrons are considered mobile and collisions and compressional effects are neglected; in this case, $\epsilon_{1,2,3}$ in (3) are real functions of ω, ω_p, ω_c). However, the refractive index surfaces still characterize the relation between $\underline{p}$ and $\underline{S}$ as above and therefore contain much pertinent information about the plane wave propagation characteristics in the medium. The n vs. θ plots also provide the means of determining the ray directions across surfaces of discontinuity arising from abrupt changes in the medium properties.[6]

A meaningful interpretation of radiation and diffraction phenomena in anisotropic regions may therefore be achieved by expressing the radiation (or diffraction) field in a form identifiable in terms of incident, reflected, refracted, diffracted, etc., rays whose progress through the medium is governed by the plane wave refractive index curves. The rigorous solution of special problems furnishes the excitation amplitudes of the various ray contributions, and their further progress can then be charted by the above-sketched ray-tracing procedure. While the ray-tracing technique is adequate to account for what may be called the geometric-optical field, it does not yield information on interface phenomena associated with surface waves, lateral waves, etc., or on transition phenomena encountered when passing from one geometric-optical wave domain to the other. These effects must likewise be assessed from a rigorous analysis of tractable prototype problems. Thus, there is a need for studying "canonical" radiation and diffraction problems in anisotropic media to provide the type of information available for similar problems in isotropic regions - both in the low-frequency (Rayleigh) and high-frequency (quasi-optic) domains. The difficulties encountered even in isotropic media in the "resonance region" intermediate between the low- and high-frequency domains are expected to be further compounded when anisotropy is present.

To gain a quantitative understanding of the problems outlined above as applied to ionized plasmas, a systematic research program was initiated under this contract. In line with the philosophy that a thorough understanding of phenomena in relatively simple idealized situations will lead to a fruitful subsequent study of more difficult problems which presently appear intractable, the anisotropic plasma employed in the analysis has so far been either of the uniaxial or of the simple gyrotropic type as in (1) and (2), respectively. Even these problems involve considerable

mathematical complexity, especially when the external magnetic field is inclined arbitrarily with respect to interfaces or perturbing objects, and much work remains to be done in this category. Results obtained from studies on this contract are summarized below.

2. Uniaxially anisotropic media

a. General remarks

It was pointed out in the Introduction that the uniaxial medium contains the simplest type of anisotropy, and that it is possible to develop a modal (guided wave) formalism for such regions by slight extensions of procedures employed in isotropic problems, provided that the optic axis is parallel to the boundaries of any perfectly conducting cylindrical object immersed in the medium. These results were established in reference 3, with the additional generalization of longitudinally variable $\underline{\epsilon}(z)$ and $\underline{\mu}(z)$. It was shown that problems involving non-variable $\underline{\epsilon}$ and $\underline{\mu}$ are transformable directly into equivalent isotropic problems, and that the radiation from elementary source distributions (such as dipoles, line sources, etc.) in an infinite, homogeneous, uniaxially anisotropic space can be expressed in closed form just as in an isotropic region. In the formulation of radiation problems, due regard must be given to a radiation condition which requires the outward flow of energy from the source domain.[7a, b] Since the directions of phase and energy propagation differ in the anisotropic medium, this condition is not equivalent to the conventional one requiring an outwardly progressing phase, and one may encounter backward wave effects where the pertinent components of the phase and energy propagation vectors are oppositely directed. It is found that the far field of a lumped source comprises radially outgoing rays but that the corresponding phase fronts generally move in some other direction which can be inferred from the refractive index plots. These statements remain true for the gyrotropic medium.

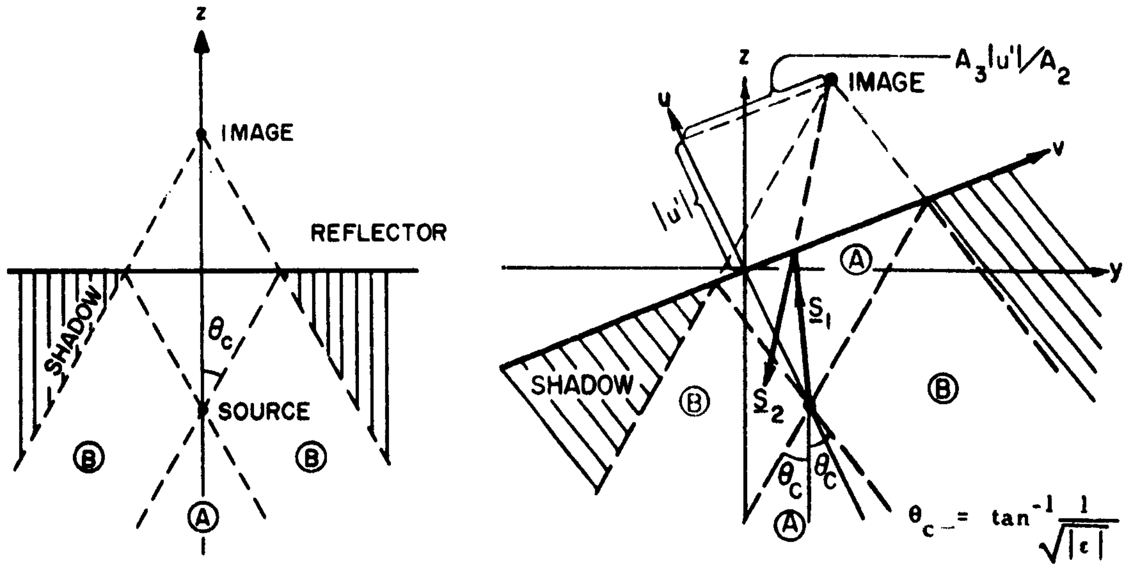
An important feature of the point source radiation field in a medium characterized by an open-branched refractive index curve as in Fig. 1(b) should be emphasized. On the ray shadow boundary $\theta = \theta_c$, the fields are found to diverge; the divergence is so strong that the real power flow through a cross-section surrounding the direction $\theta = \theta_c$ is also infinite. While this anomaly is removed if dissipation is present, its resolution for the lossless case is a matter of some interest and was actually studied first on this contract in connection with the more general problem of radiation in a plasma subjected to a finite external magnetic field (see (3)).[7a, c] It was found that the above-mentioned singularities are due to source distributions

whose strength drops to zero abruptly, and that the fields are bounded if the source intensity decays gradually. Thus, by shaping the source distribution, it is possible to focus energy along the direction of the shadow boundary cone. The radiation patterns of various source distributions are now under study.

From the preceding remarks, it is noted that the formal solutions for radiation from sources in the presence of plane interfaces perpendicular to z, or of cylinders, wedges, half-planes, etc., parallel to z, can be constructed with comparative ease for the uniaxial medium. The real task is an asymptotic evaluation and meaningful physical interpretation of these results. Such a study is in progress and has been completed so far for the case of radiation from a plasma half-space.

b. Radiation in the presence of a perfect reflector

If the refractive index surface has an unbounded branch as in Fig. 1(b), the propagating rays due to a longitudinal electric current source are confined to the interior of the cone $\theta < \theta_c$, where θ is measured from the dipole axis, and the region $\theta > \theta_c$ is opaque and supports only exponentially decaying waves. The extent of the shadow zone can be reduced by insertion of a reflector (Fig. 2) which permits the penetration of the reflected rays into part of the shadow region of the source. If the reflector is oriented perpendicular to the z-axis, its effect can be rigorously accounted



(a) reflector perpendicular to z-axis (b) reflector inclined to z-axis

Fig. 2 - Plasma half-space bounded by perfect reflector - Rays from the source penetrate region A directly but region B only by reflection.

for in terms of an image source as in Fig. 2(a).[3] For an inclined reflector, the analysis is more complicated and has been carried out so far only for the two-dimensional problem of excitation by a line source of magnetic currents directed transverse to z (say, along the x-axis). In this case (Fig. 2(b)), the reflected rays again appear to come from an image source which is, however, displaced from the mirror image point by the amount $A_3|u'|/A_2$, where $A_2 = \cos^2\alpha + (1/\varepsilon)\sin^2\alpha$, $A_3 = \left[1 - (1/\varepsilon)\right]\sin 2\alpha$, $\varepsilon = \varepsilon_2/\varepsilon_1$, α is the angle between the reflecting plane and the y-axis, and $|u'|$ is the distance from the source to the reflector at $u = 0$. These results are proved via two different methods in references 8 and 9.

c. Radiation from a half-space

A detailed study of radiation from a uniaxially anisotropic plasma half-space was carried out for the cases where the interface is either perpendicular to the z-axis[3,10] or inclined arbitrarily.[8] For the former, arbitrary source distributions are admitted; the latter analysis has been performed so far only for the two-dimensional problem of magnetic line source excitation. An asymptotic evaluation of the formal representation integrals by the steepest descent method reveals that the far field in the interior or exterior of the plasma half-space comprises contributions which can be identified as direct, reflected or refracted rays with the reflection and refraction laws exactly as predicted from the refractive index curves (the reflected field can also be thought of as arising from an image source as in Fig. 2). Thus, it is verified that these fields can be predicted by ray-tracing methods, once their starting amplitude is known.

However, there exists an additional field contribution due to a branch cut integral in the asymptotic formulation which can be interpreted as a new type of lateral wave; this wave propagates parallel to the interface in the exterior region, sheds energy into the plasma by critical refraction and is itself excited by a ray incident from the source at the critical angle. For the medium having a refractive index curve as in Fig. 1(b), two lateral waves exist and their ray paths are shown in Fig. 3. The direction of the various rays can again be predicted from Fig. 2(b). Most significant is the fact that the lateral waves penetrate the shadow regions in the plasma which are inaccessible to either the direct or reflected rays. While the lateral wave fields in the illuminated region are generally smaller than the geometric-optical fields, they constitute the dominant contribution in the shadow zone in which the geometric-optical solutions are exponentially small. Thus, lateral waves may be expected to play a more important role on interfaces bounding anisotropic media than on isotropic boundaries; their effect on the latter has been studied in some detail in the literature, especially in connection with pulse propagation.[11]

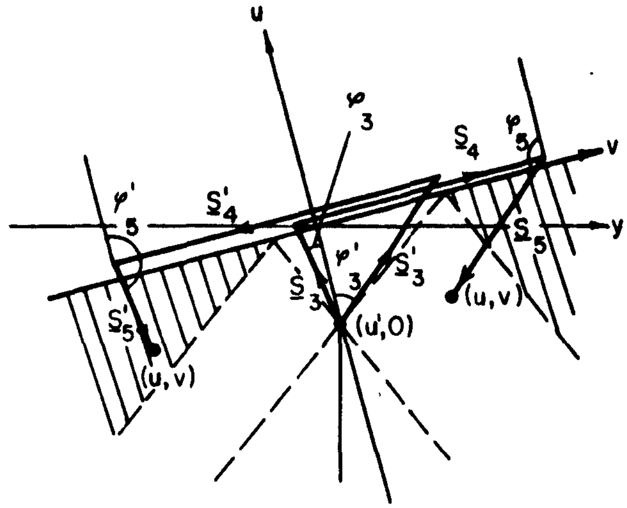


Fig. 3 - Lateral ray paths

The simple description above fails in the vicinity of the transition regions delimiting the various ray-optical domains, and the more complicated field calculations required there have likewise been carried out.[8] In this connection, results were derived for the asymptotic evaluation of integrals possessing three evenly spaced neighboring saddle points.

It is apparent from the above and should be emphasized that the rays, and not the phase gradients, play the significant role in the physical interpretation of radiation and diffraction phenomena in anisotropic media.

d. Ring source and plasma column

An analysis of the radiation from a ring source of magnetic currents concentric with a solid or hollow plasma column inside a circular waveguide has been carried out. The column is subjected to either zero or infinite axial magnetic field; in the latter instance, it becomes uniaxially anisotropic. The purpose of this study was to determine the amplitude of excitation of slow E-type surface waves which can be guided either by the plasma - air interface or by the plasma interior; the results are of interest for applications involving interaction with longitudinally injected electron beams. The eigenvalue (transverse resonance) problem was studied in detail and numerical calculations for the possible surface waves of interest, as well as their amplitudes of excitation, have been presented in reference 12.

e. Diffraction by objects oriented perpendicular to the optic axis

While the remarks in Sec. (a) have been pertinent for the formulation of diffraction problems when a cylindrical scatterer (of arbitrary cross section) is oriented parallel to the optic (z) axis, another interesting class involves perpendicular orientation. For a curved obstacle, e.g., a circular cylinder, the direction of the optic axis then varies over the surface, thereby giving rise to effects not encountered previously. One may show[9] that two-dimensional problems of excitation by a line source of magnetic currents parallel to the obstacle axis are transformable into equivalent isotropic problems, in which one of the coordinates of the given configuration is "stretched" according to $y' = \sqrt{\epsilon}\, y$. Thus, a circular cylinder in the anisotropic medium becomes equivalent to an elliptic cylinder in an isotropic medium; an inclined plane surface transforms into a plane surface inclined at a different angle; etc. As in the previously mentioned class of problems, the major task here is not the construction of formal solutions but rather their meaningful physical interpretation. Some preliminary results involving a plane reactive surface, and a perfectly conducting wedge have been given in reference 9.

3. Plasma under the influence of a finite external magnetic field

a. Network formulation

The simplest model of a plasma under the influence of a z-directed finite external magnetic field involves the dielectric tensor (3) which is Hermitean if losses are neglected. The solution of the source-dependent Maxwell field equations in this tensor medium poses considerable difficulties even for the infinite homogeneous space, to say nothing of inhomogeneous or bounded regions, or regions containing scattering objects.* A thorough study of radiation from arbitrary source distributions in transversely unbounded anisotropic media with plane stratification along the .-direction was undertaken.[7a-c] While a general, abstract formulation of problems of this type has been available[13], it was found desirable to proceed in the present case by a generalization of modal procedures familiar from the analysis of plane stratified isotropic regions. The Maxwell field equations were reduced to their transverse form, the transverse (to z) fields expressed in terms of a continuous superposition of orthogonal vector modes which comprise both "ordinary" and "extraordinary" constituents (see (4)), and the modal amplitudes derived from the solutions of conventional transmission line equations (5). In this manner, the field problem is reduced to a modal network problem

*Simplifications arise for certain two-dimensional problems in which the fields have no z variation.

involving source-excited transmission lines, with the continuity requirements of the transverse fields across a plane interface separating two different media expressed in terms of a network which couples the ordinary and extraordinary modes (see Fig. 4). These coupling networks have also been given for plane boundaries with constant surface impedance. While the analysis is conceptually simple, the actual expressions for the mode functions and especially for the network elements are quite involved; they have been listed in reference 7a.

The resulting representation for the electric field $\underline{E}_t$ has the form

$$\underline{E}_t(x, y, z) = \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta \left[V_o(z;\xi, \eta)\underline{e}_o(\xi, \eta) + V_e(z;\xi, \eta)\underline{e}_e(\xi, \eta) \right] e^{i(\xi x + \eta y)} \quad , \quad (4)$$

where $\underline{e}_o$ and $\underline{e}_e$ are the ordinary and extraordinary mode eigenvectors, respectively. V_o and V_e, the corresponding mode amplitudes, are determined from the transmission line equations

$$\frac{dV}{dz} = i\kappa ZI + \hat{v} \quad , \quad \frac{dI}{dz} = i\kappa YV + \hat{i} \quad , \quad Z = \frac{1}{Y} \quad , \quad (5)$$

which hold separately for each mode and in each of the constituent media. $\hat{v}$ and $\hat{i}$ are voltage and current sources whose strengths are known in terms of the applied electric and magnetic current distribution. $\kappa = \kappa(\xi, \eta)$ is the modal propagation constant and Z the characteristic impedance. $\underline{e}_{o,e}$, $V_{o,e}$, $\kappa_{o,e}$ and $Z_{o,e}$ generally are multi-valued functions of $\sigma = \sqrt{\xi^2 + \eta^2}$, and their proper specification poses one of the problems in rendering the integral representation (4) unique. A similar formulation exists for the transverse magnetic field $\underline{H}_t$.

b. The radiation fields in an infinite medium

The preceding analysis permits the systematic construction of the formal solutions for the fields radiated by arbitrary, but prescribed, sources in a plasma medium consisting of piecewise constant layers along the z-direction. If the region extends to infinity along z, a radiation condition must be imposed which, as noted on p. 5, requires an outward flow of energy. The above-mentioned multivaluedness of the propagation constant κ is intimately connected with the radiation condition, and both an analytical[7a] and a graphical[7b] procedure have been given for its resolution. Because of the exceedingly complicated analytical structure of the integrand in (4), only two problems have been studied in detail at this time: a) radiation in an infinite, homogeneous medium, and b) radiation from a half-space. While in an initial attempt

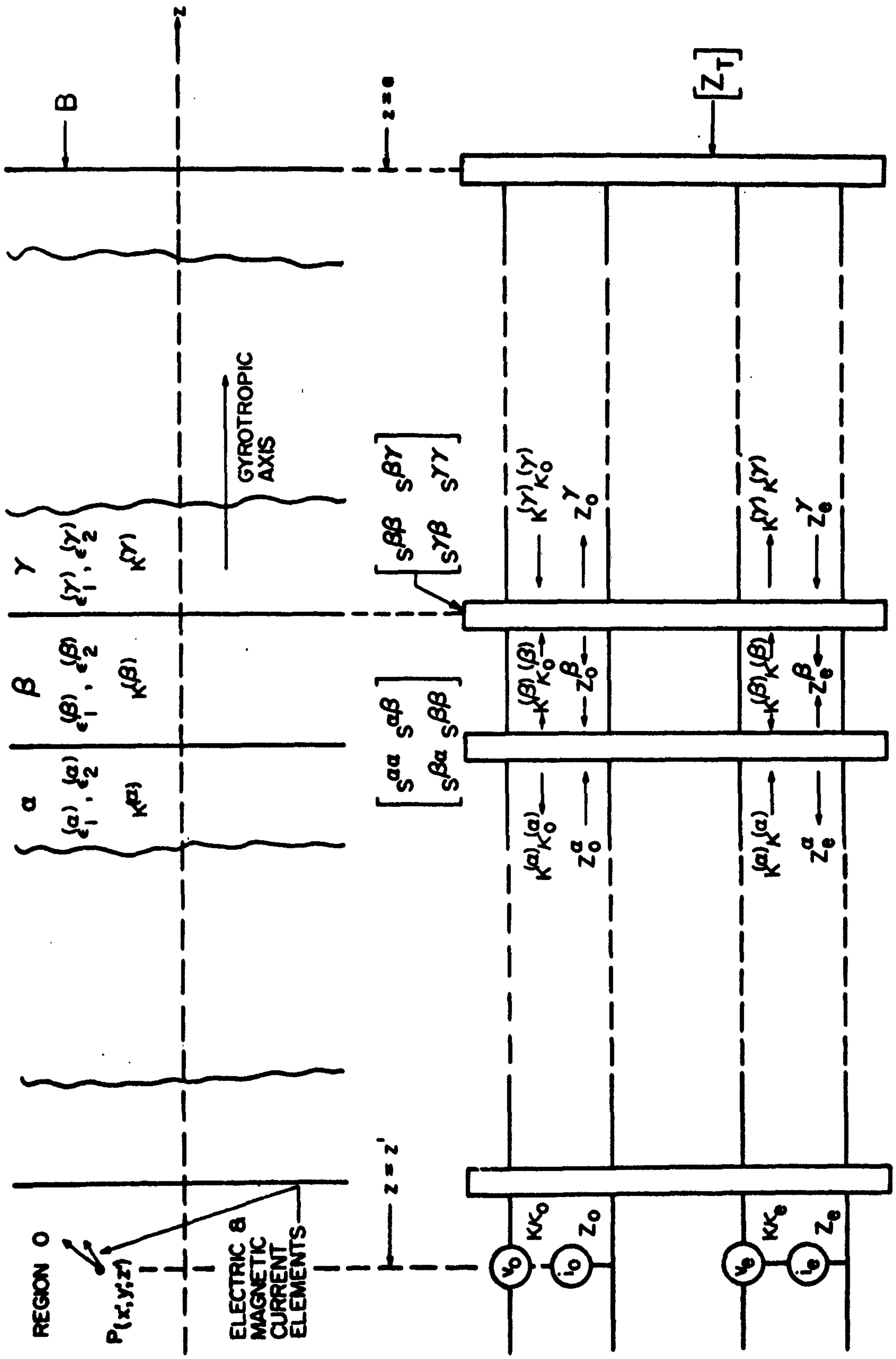


FIG. 4. THE PHYSICAL STRUCTURE AND THE ASSOCIATED NETWORK PROBLEM (LONGITUDINAL GYROTROPIC AXIS)

at reducing the integrals, a perturbation procedure appropriate to weak external magnetic field H_o was employed[14], the final analysis was carried out for the general case[7a-c] and the results are summarized below.

The asymptotic evaluation of the fields radiated by a dipole source in the infinite medium was carried out by a saddle point procedure. Only real saddle points which yield propagating wave contributions were taken into account; complex saddle points give rise to exponentially decaying fields. The number of real saddle points is found to depend on both the medium properties and on the observation point location, and is predictable from the refractive index curves for the medium. For a given observation point located at an angle θ in the ρ-z plane, the saddle points correspond to those points on the refractive index surface from which the surface normals also point in the θ direction. If the n vs. θ surface consists of two separate branches as in Fig. 5, for example, there may be as many as four saddle points, and each contributes a propagating ray from the source to the observation point. These rays individually carry

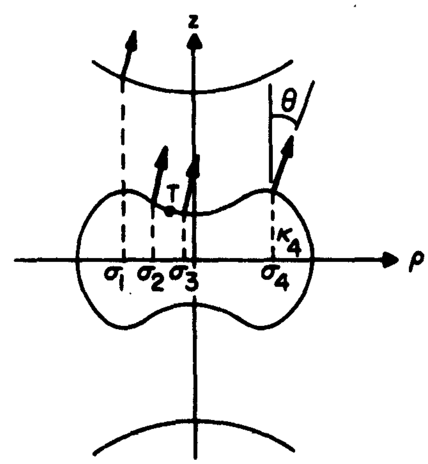


Fig. 5 - Graphical interpretation of saddle point condition

energy in the radial direction, but due to their different phase velocities, interference results and the total power flow need not be radial. These interference effects may perhaps be helpful in connection with plasma diagnostics. The ray amplitudes decay inversely with distance from the source as in a spherical wave, and they are also found to be proportional to $\sqrt{R_i}$, where R_i is the radius of curvature of the refractive index curve at the i-th saddle point. If the rays correspond to a point of inflection T, $R_i \to \infty$ and two saddle points coalesce. The necessary asymptotic evaluation then

requires simultaneous consideration of two neighboring saddle points, and the fields are represented in terms of Airy functions rather than exponentials. R_1 also diverges when the saddle point moves to infinity along an unbounded branch of the $n(\theta)$ curve. In that case, the fields exhibit true singularities* which can, however, be removed upon replacing the delta function (point) source by a source distribution of gradually diminishing strength (see comments on p.7). Near the singular directions where a ray is perpendicular to an asymptote of the open-branched surface, the divergent part in the integral representation for the fields may be shown to be the same as for the uniaxial medium which can be expressed in closed form; this recognition permits the detailed study of the fields in the vicinity of its singularities.

The results of this analysis have been applied to the calculation of the radiation patterns of a longitudinal electric dipole source in a gyrotropic plasma. All possible ranges of ω/ω_p and ω/ω_c have been considered and a typical calculation has been performed for each set of parameters which gives rise to a characteristically different refractive index diagram. The numerical work was carried out on an electronic computer and the resulting curves are given in references 7a, c.** The great diversity in the radiation patterns shows clearly the dependence of the radiation field on the medium parameters ω_p, ω_c, and on the applied frequency ω.

c. Radiation in the presence of a plasma half-space

The preceding theory has also been applied to the analysis of the radiation from a z-directed electric current element in the interior or exterior of a longitudinally magnetized plasma half-space bounded by the plane surface z = 0. The integrands in (4) are now much more complicated than for the infinite medium, and the asymptotic calculation of the fields has been carried out only for the geometric-optical constituents, thereby ignoring such possible additional contributions as surface waves or lateral waves (arising from poles and branch points, respectively), which may be important near the interface or in shadow regions. A complete ray-optical interpretation of the

*These phenomena do not necessarily occur in an actual plasma whose representation by the simple tensor (3) is valid only in the linear, small signal approximation. However, increased field strengths in a plasma do most likely exist near the singular directions noted above.

**These calculations were performed by E. Arbel in connection with his doctoral dissertation. A revised and condensed version of the first part of the thesis was intended to be published jointly by E. Arbel and L. B. Felsen. Due to Arbel's departure to his native country of Israel, this revision was delayed and it was decided to issue the thesis in its original form (reference 7a) in order to make its contents available. The revised versions have now been completed (references 7b, c) and will be issued shortly.

asymptotic solutions for the reflected and refracted fields has been given [7a] and it has been shown how the refractive index curves can be used to establish the ray trajectories. Under certain conditions one may encounter such phenomena as backward refraction and conical refraction.

The methods of analysis employed in the anisotropic medium studies on this contract, and the results achieved, have been compared in references 7b, c with related work by other authors which has become available during the period of this Contract. Almost all of the other investigations have proceeded from a three-dimensional Fourier integral approach rather than from the two-dimensional modal technique leading to the double Fourier integrals in (4). While the triple Fourier integral procedure has a certain formal elegance, the derivation of asymptotic results in an infinite medium, or even the formal analysis of plane stratified media, requires its eventual reduction to the two-dimensional integral from which is obtained at once from the modal (network) approach. Especially in connection with the plane stratified medium problem, the network procedure offers a systematization which is lacking in other methods. These advantages have recently been pointed out by Wu[15] in his analysis of radiation in the presence of an anisotropic plasma slab.

4. Modes in a hydromagnetic waveguide

J. Shmoys

Modes in a hydromagnetic waveguide have been investigated by Newcomb[16]. Newcomb finds two sets of modes in a waveguide of circular cross-section containing a low-pressure perfectly conducting fluid in an axial dc magnetic field at a pressure that is much smaller than the magnetic field pressure $B^2/2\mu_o$. He then finds how these modes are perturbed (to first order) by taking into account "Ohm's law" in two different forms (with a gyration term and with a conduction term) and by assuming the pressure to be sufficiently low to make the speed of sound small compared with the velocity of the magnetohydrodynamic waves. The two sets of modes are called transverse electric and principal. Furthermore, the author remarks: "There is nothing analogous to the TM modes of an optical waveguide, because these would have a longitudinal electric field."

We have shown[17] that the equations governing the behavior of the electromagnetic field in an imperfectly conducting, perfectly tenuous fluid can be described by Maxwell's equations for a uniaxial medium whose dielectric constant in the longitudinal direction is $(\epsilon_o + \sigma/j\omega)$, where ϵ_o is the dielectric constant of free space, σ

is the conductivity of the fluid, and ω the wave frequency, while the transverse dielectric constant is $\left(\varepsilon_o + \frac{1}{(j\omega/\sigma) + B_o^2/\rho}\right)$, where B_o is the applied dc magnetic flux density and ρ is the density of the medium. The modes in a waveguide filled with a uniaxial medium do fall into the sets (TE and TM), but in this case, in the limit of infinite conductivity the longitudinal electric field of the TM modes tends to zero. The transverse field dependence of the various modes is identical with that in a waveguide filled with isotropic dielectric (see also Sec. 2).

References

1. cf. K.G. Budden, "Radio Waves in the Ionosphere", Cambridge Univ. Press (1961).

2. cf. P.J.B. Clarricoats, "Microwave Ferrites", Wiley (1961).

3. E. Arbel and L.B. Felsen, "On Electromagnetic Green's Functions for Uniaxially Anisotropic Regions", Report PIBMRI-985-61, Microwave Res. Inst., Polytech. Inst. of Brooklyn, Feb. 1962.

4. reference 1, Sec. 13.18.

5. P.C. Clemmow and F. Mullaly, "The Dependence of the Refractive Index in the Magneto-Ionic Theory on the Direction of the Wave Normal", The Physics of the Ionosphere, The Physical Society, London (1954), p. 340.

6. reference 1, Sec. 13.21.

7a. E. Arbel, "Radiation from a Point Source in an Anisotropic Medium", Report PIBMRI-861-60, Microwave Res. Inst., Polytech. Inst. of Brooklyn, Nov. 1960.

b. E. Arbel and L.B. Felsen, "On the Theory of Radiation from Sources in Plane Stratified Anisotropic Media", Report PIBMRI-1069-62, Microwave Res. Inst., Polytech. Inst. of Brooklyn, Sept. 1962.

c. E. Arbel and L.B. Felsen, "Radiation from a Dipole in an Infinite Homogeneous, Anisotropic Plasma", Report PIBMRI-1070-62, Microwave Res. Inst., Polytech. Inst. of Brooklyn, Sept. 1962.

8. L.B. Felsen, "Radiation from a Uniaxially Anisotropic Plasma Half-Space", Report PIBMRI-1058-62, Microwave Res. Inst., Polytech. Inst. of Brooklyn, Aug. 1962.

9. L.B. Felsen, "On Diffraction by Objects in a Uniaxially Anisotropic Medium", Electrophysics Memorandum No. 83, PIBMRI-1073-62, Microwave Res. Inst., Polytech. Inst. of Brooklyn, Aug. 1962.

10. L. B. Felsen, "Lateral Waves on an Anisotropic Plasma Interface", Electrophysics Memorandum No. 74, PIBMRI-974-61, Microwave Res. Inst., Polytech. Inst. of Brooklyn, Jan. 1962.

11. cf. L. M. Brekhovskikh, "Waves in Layered Media", Academic Press, New York (1960), Sec. 21.

12. S. N. Samaddar, "Study of Surface Waves Along the Boundary of Two Coaxial Cylindrical Columns in a Circular Cylindrical Metallic Waveguide with Excitation by a Magnetic Ring Current", Report PIBMRI-857-60, Microwave Res. Inst., Polytech. Inst. of Brooklyn, Oct. 1960.

13. A. D. Bresler and N. Marcuvitz, "Operator Methods in Electromagnetic Theory", Report MRI-495-56, Microwave Res. Inst., Polytech. Inst. of Brooklyn, 1956.

14. 7th Quarterly Report issued on this Contract, May 19, 1960.

15. C. P. Wu, "A Study on the Radiation from Elementary Dipoles in a Magneto-Ionic Medium", Report 1021-20, Dept. of Elect. Eng., Ohio State U., Aug. 1962.

16. W. A. Newcomb, "The Hydromagnetic Wave Guide", in Magnetohydrodynamics: A Symposium, edited by R. K. M. Landshoff (Stanford Univ. Press, 1957).

17. J. Shmoys and E. Mishkin, "Hydromagnetic Waveguide with Finite Conductivity and Arbitrary Cross-Section", Phys. of Fluids, Vol. 3, No. 3, 1960.

C. Propagation and Diffraction Problems in Isotropic Plasmas

1. Piecewise homogeneous regions

a. The electromagnetic field of a source-excited plasma layer

T. Tamir and A. A. Oliner

The main scope of this research effort was the investigation of the electromagnetic properties of a planar homogeneous, isotropic plasma slab, including problems of radiation through plasma layers from finite sources. The purpose of this study was to clarify via this idealized model the importance and influence of plasma sheaths which surround a missile upon its re-entry in the upper atmosphere, and thus to obtain a proper understanding of the effect that such an ionized layer will have on the communication to and from the re-entering vehicle.

The major aspects of this investigation may be classified into the following three topics:

1. The electromagnetic spectrum of an infinite plasma slab.
2. The near field and radiation pattern of a slot-excited plasma slab.
3. Radiation from semi-infinite slot-excited plasma sheath configurations.

The principal features of each of these studies are described separately below.

1'. The electromagnetic spectrum of an infinite plasma slab

This part of the study[1, 2, 4] consists of an extensive investigation of all the types of waves which may be supported by a lossless and homogeneous plasma layer. The geometry of the configuration is shown in Fig. 6; the plasma slab is characterized by a relative dielectric constant:

$$\epsilon_p = 1 - \left(\frac{\omega_p}{\omega} \right)^2 \tag{1}$$

where ω_p is the plasma frequency and ω is the frequency of the applied field.

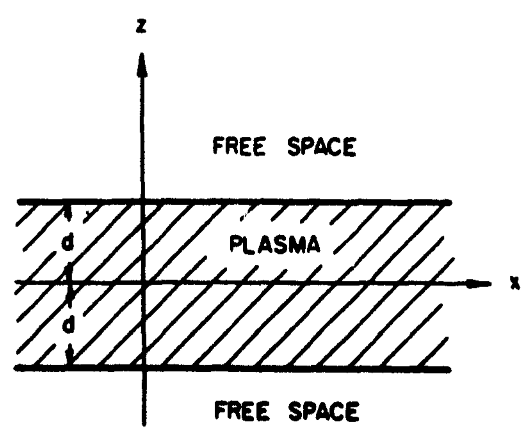


Fig. 6 - Geometry of the homogeneous isotropic plasma slab

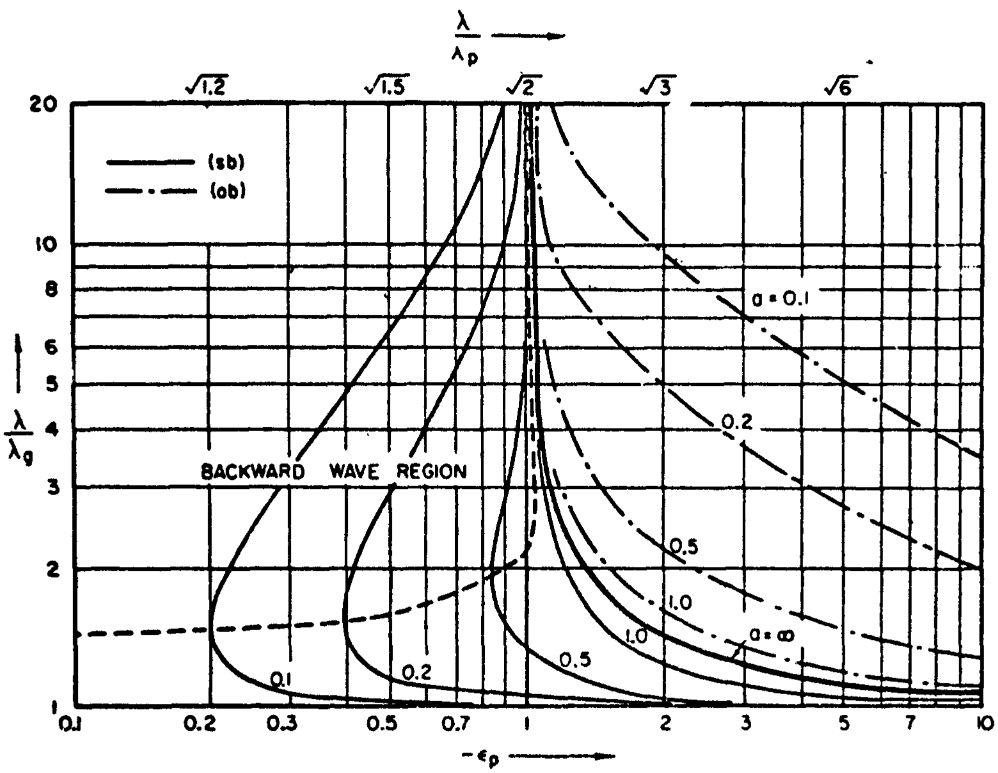


Fig. 7 - Dispersion curves for surface waves along the plasma slab; λ/λ_g is shown versus either ϵ_p or λ/λ_p for both the short-circuit (sb) and open-circuit (ob) bisection cases, for different values of $a = 2\pi d/\lambda_p$, with:

λ: wavelength in free space,

λ_g: guided wavelength along the plasma slab,

$\lambda_p = 2\pi c/\omega_p$,

c: velocity of light in free space.

The dotted line shows the boundary between the forward wave and the backward wave regions.

In order to find any possible discrete contributions to the spectrum, the problem was analyzed by assuming either an electric or a magnetic plate at the plane of symmetry $z = 0$; these solutions are then referred to, respectively, as short-circuit (sb) or open-circuit (ob) bisection solutions.

It was then found that E mode surface waves may be present along the slab whenever ε_p is negative, as shown by the dispersion curves of Fig. 7. The field of these surface waves decays away from the plasma-air interface in both the plasma and air regions. An interesting feature of these waves is the fact that, for some values of the parameters involved, the waves may be of the backward type, as indicated in Fig. 7. This feature is novel for isotropic configurations and it is noted that the surface waves discussed here are independent modes and not backward space-harmonics such as occur in periodic structures.

In addition to these surface waves, other modes with complex (rather than pure real) wavenumbers were shown to be present for E modes and for negative values of ε_p. These waves, unlike improper (non-modal) leaky waves, possess fields which are defined everywhere in space and decay in all directions away from the source. The field contours for this type of wave are shown in Fig. 8, which refers to the case of a short-circuit (sb) bisected slab. Although, as indicated in this figure, power flow seems to be associated with such a complex wave, actually two such waves must be present simultaneously in lossless media and they have their power flow in opposite directions; the net result of the existence of both waves is then a cancellation of the total real power flow, with only reactive power being left in the form of stored energy in the neighborhood of the source.

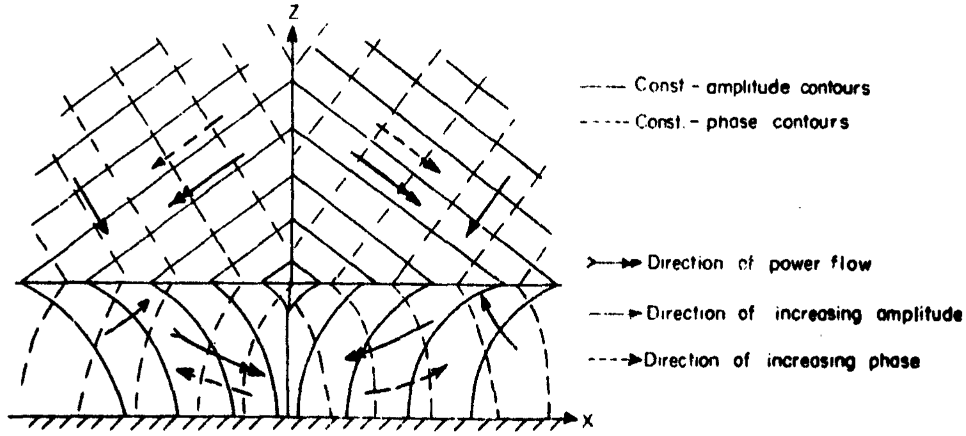


Fig. 8 - Field contours for a complex spectral wave

When E modes with $\epsilon_p > 0$ or H modes (with any value of ϵ_p) were considered, it was shown that no surface or spectral complex waves can exist. Instead, by using a non-spectral representation, it was shown that improper leaky waves may be excited; these leaky waves have a strong influence on the shape of the radiation pattern due to a source located in or near the slab. This aspect is discussed in the next section.

2'. The near field and radiation pattern of a slot-excited plasma slab

The information obtained from the study described above was used[3, 6] to find the radiation pattern of a plasma layer backed by a metal plate when the excitation is in the form of a slot located in the plate, as shown in Fig. 9, where the slot is represented by a magnetic line source. This particular geometry was chosen since it corresponds to a first idealized model of an antenna located on the wall of a missile when the latter is enclosed by a plasma sheath.

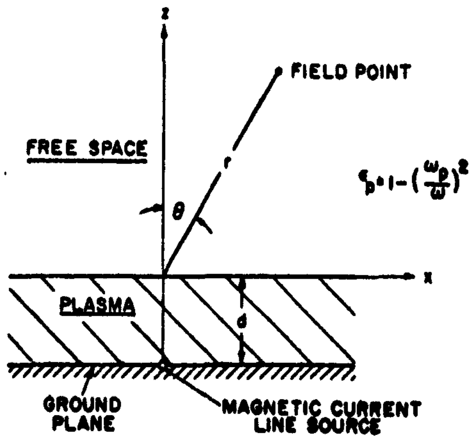


Fig. 9 - Geometry of plasma layer

It was then shown that, when $\epsilon_p > 0$, the near field is dominated by the presence of a single leaky wave. The air-plasma interface was then chosen as an aperture surface, and the leaky wave along this surface taken as the aperture distribution, in order to compute the associated radiation pattern. The resulting pattern exhibited a sharp major peak, as expected from previous calculations of such patterns obtained by steepest descent techniques (in the absence of the plasma layer, the radiation pattern is, of course, isotropic with respect to the polar angle θ). This major peak is present at an angle θ_o given closely by:

$$\sin \theta_o = \sqrt{\epsilon_p} \quad . \tag{2}$$

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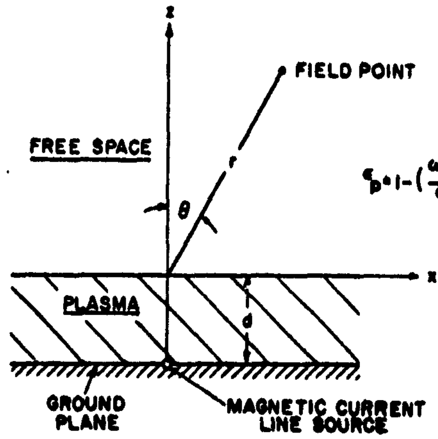

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$$\sin \theta_o = \sqrt{\epsilon_p} \quad . \qquad (2)$$

This angle θ_o is the result which one would expect from geometric-optical arguments; however, it also corresponds to the radiation from the leaky wave which is most strongly excited. In fact, it may be shown that, in most cases, the radiation pattern can be found by assuming the field at the plasma-air interface to consist only of the contribution due to this leaky wave. This is exemplified by the radiation patterns shown in Fig. 10 where $R_s(\theta)$ is the asymptotically rigorous (steepest descent) result for the radiation pattern, while $R_p(\theta)$ is the result obtained when one assumes the near field due only to the major contributing leaky wave. One then observes that other field contributions (from the space wave and/or minor leaky waves) yield only small corrections to $R_p(\theta)$.

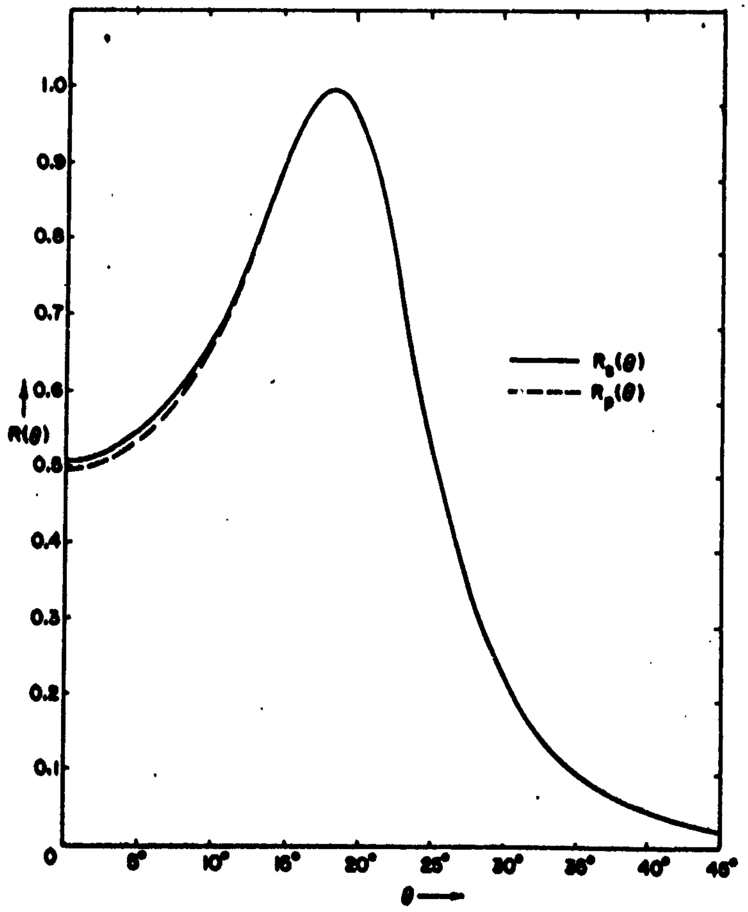

Fig. 10 - Radiation pattern for $a = 1.0$ and $\epsilon_p = 0.1 (d/\lambda = 0.168)$. The first leaky wave is strongly dominant; the other leaky waves yield broadside radiation only.

When the slab thickness d is relatively large, other minor leaky waves yield additional smaller peaks, as shown in Fig. 11. In that case, the locations of the peaks may also be predicted and are associated with the presence of these latter peaks, as indicated in the figure. It then follows that, in general, the features of the radiation pattern can be directly related to and easily predicted from the values of the wavenumbers of the leaky waves. When one calculates these wavenumbers as functions of the frequency of operation, the thickness of the slab or the electron concentration in the plasma, one can easily find how some of the major features of the radiation pattern change when any one of these parameters varies.

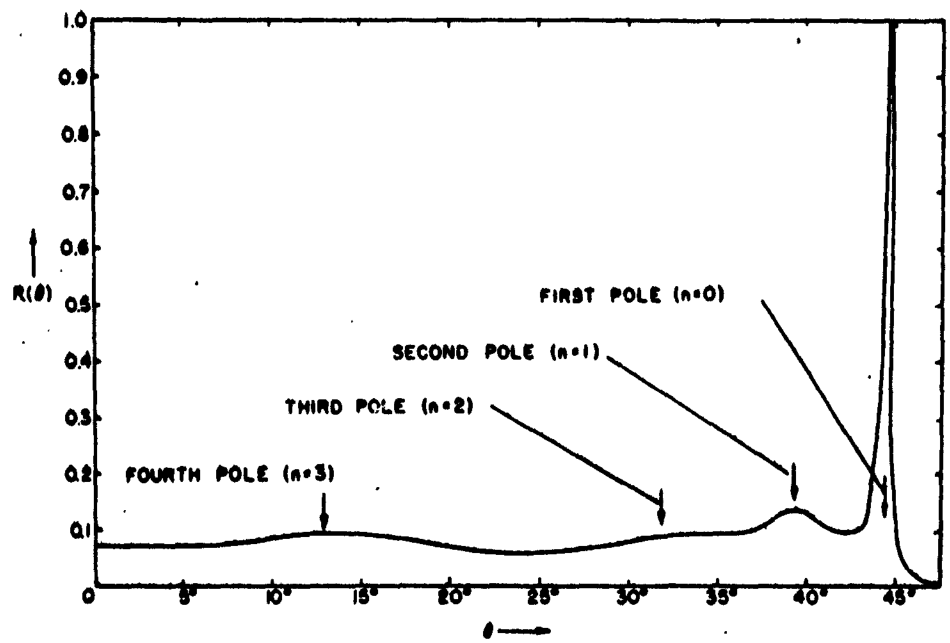

Fig. 11 - Radiation pattern for a = 10.0 and ϵ_p = 0.5 (d/λ = 1.414). The arrows show the angles of maximum radiation for the first four leaky waves; all the other leaky waves contribute at broadside only.

When $\epsilon_p < 0$, the field in the plasma medium decays exponentially so that very little radiation is present. Then, energy may be carried along the slab due to a surface wave and a small amount of broadside radiation is present; the latter may be shown to be associated with the presence of the spectral complex poles mentioned in Sec. A.

3'. Radiation from semi-infinite slot-excited plasma sheath configurations

Since a missile is in fact finite and not infinite in extent, an improvement over the model considered in Sec. B is obtained by taking into account the influence of finiteness or of discontinuities which may be present.[5] In particular, the two configurations shown in Fig. 12 were considered.

In the first case, Fig. 12(a), the plasma slab is terminated at a plane AA' to approximate the effect of the electron concentration thinning out to negligible amounts. In the second case, Fig. 12(b), the presence of the end wall of the missile is accounted for by letting the plasma layer continue and fill the quarter-space vacated by the metal.

Since the near field is dominated by a single leaky wave, it was assumed that the field at the discontinuity plane AA' is given by this leaky wave alone, for both cases shown in Fig. 12. The radiation field was then calculated by means of a Kirchhoff integration over this field, and the higher mode effects at the discontinuity were disregarded. The result thus obtained was evaluated numerically for many values of the parameters involved. In all these cases, it was found that the major radiation peak discussed in Sec. B is present for the semi-infinite case as well; other peaks may then also appear but they are small in amplitude in most cases. In addition, some end-fire ($\theta = \pi/2$) radiation is now present.

Most notably, however, it was found that the difference between the semi-infinite and infinite cases was strongest when the source was located close to the discontinuity plane AA'. When the separation between the source and plane AA' is only one or two wavelengths, the pattern is strongly modified; when this separation is greater than 10 or 20 wavelengths, the pattern is indistinguishable from that for the infinite case.

These results therefore show that, except for some minor changes and except when the source is very near to the discontinuity, the radiation patterns for these cases are not appreciably different from those for the infinite case. One therefore concludes that the major peak due to the most strongly excited leaky wave is a feature which dominates the radiation pattern and, as such, is not much affected by the presence of discontinuities as long as these are not located too close to the source.

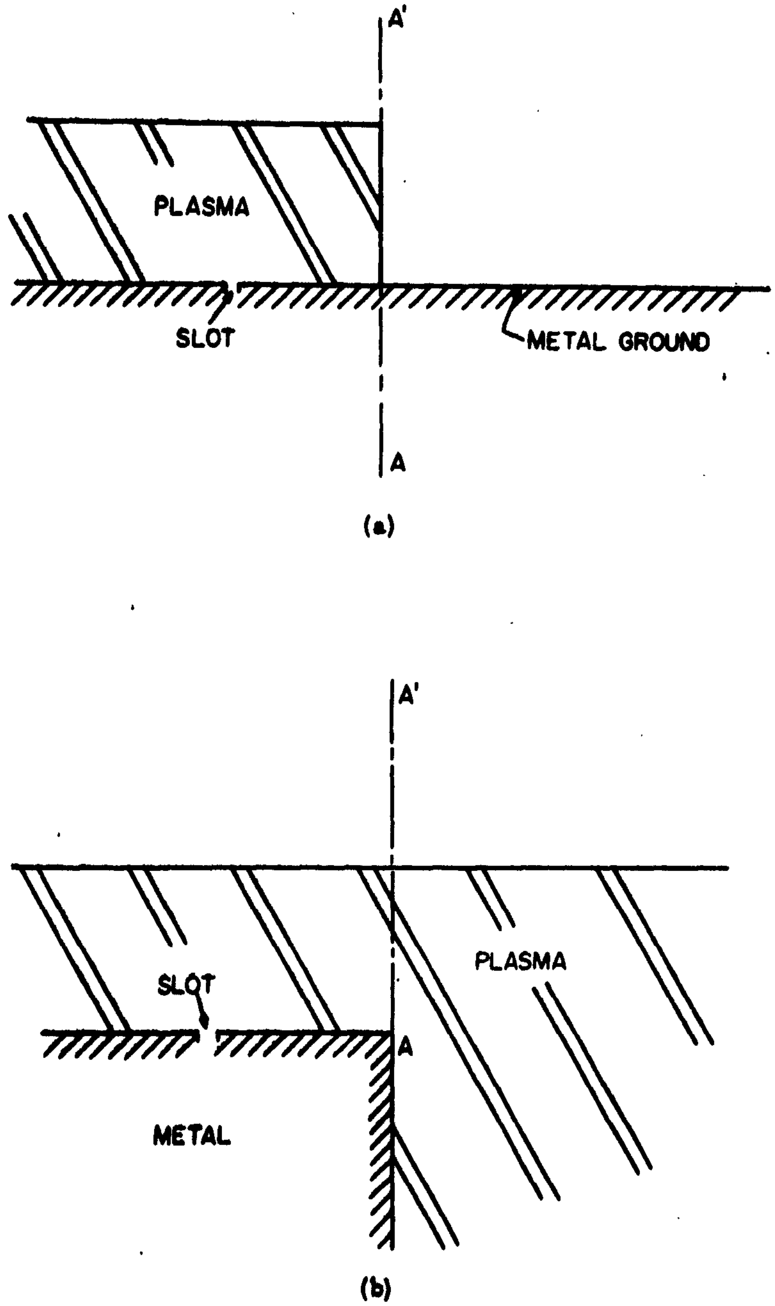

Fig. 12 - Geometries for the semi-infinite plasma layer

<u>References</u>

1. A. A. Oliner and T. Tamir, "Backward Waves on Isotropic Plasma Slabs", Electrophys. Group Memo. 66, PIBMRI-930-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, 1961.

2. T. Tamir, "The Electromagnetic Field of a Source-Excited Plasma Slab", PIBMRI-932-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 1961.

3. T. Tamir and A. A. Oliner, "The Influence of Complex Waves on the Radiation Field of a Slot-Excited Plasma Slab", PIBMRI-959-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Oct. 1961.

4. T. Tamir and A. A. Oliner, "The Spectrum of Electromagnetic Waves Guided by a Plasma Layer", PIBMRI-970-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Dec. 1961.

5. A. A. Oliner and T. Tamir, "Radiation from Semi-Infinite Slot-Excited Plasma Sheath Configurations", PIBMRI-1029-62, Microw. Res. Inst., Polytech. Inst. of Brooklyn, May 1962.

6. T. Tamir and A. A. Oliner, "The Near Field Supported by a Source-Excited Plasma Slab", URSI-IRE, May 1961, Washington, D. C.

b. Radiation and propagation in the presence of a compressible plasma half-space

J. Shmoys

Most of the existing work dealing with the propagation of electromagnetic waves on plasma structures is based on the assumption that only electrons are mobile and that the electron temperature is zero. This investigation deals explicitly with the effect of finite electron temperature. At a finite temperature an electron gas can propagate longitudinal waves with a finite velocity. If it were possible to affect the radiation of electromagnetic waves to an appreciable extent by changing plasma temperature, then the manner in which this influence occurs would be of interest both from the point of view of the performance of electromagnetic devices in the presence of warm plasmas and from the point of view of possible diagnostic applications.

The structure investigated here was a semi-infinite plasma bounded from the rest of space by a stationary plane interface. A magnetic line current parallel to the interface, located in free space at a finite distance from the boundary, was found to excite plasma waves as well as a surface wave. This surface wave also exists in the same configuration having zero electron temperature, and the electron temperature affects its properties to some extent. Early in this investigation[1] it was erroneously concluded that as a result of the finiteness of temperature, a second surface wave could propagate; it was later found that this second root of the resonance equation does not lie on the proper sheet of the appropriate Riemann surface, and thus does not correspond to a surface wave. At zero temperature the surface wave can propagate only at frequencies which are smaller than the plasma frequency by a factor of at least $\sqrt{2}$, i.e., $\omega < \omega_p / \sqrt{2}$. The effect of finite temperature is to extend the pass-band of the surface wave to infinite frequencies. At low frequencies, below the zero temperature cut-off frequency, temperature has little effect on the propagation of the surface wave. At higher frequencies, the dispersion curve becomes very steep so that the surface wave will hardly be excited. Almost all the power carried by the surface wave is in the electromagnetic field. A calculation of the total power transfer into the longitudinal oscillations is still in progress.

The results of this investigation were issued in a report[2] and published[3].

1. 9th and 12th Quarterly Reports.
2. A. Hessel, N. Marcuvitz and J. Shmoys, "Scattering and Guided Waves at an Interface Between Air and a Compressible Plasma", PIBMRI-953-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 1961.
3. A. Hessel, N. Marcuvitz and J. Shmoys, "Scattering and Guided Waves at an Interface Between Air and a Compressible Plasma", IRE Trans. PGAP, Vol. AP-10, pp. 48-54, Jan. 1962.

c. Surface waves on plasma transition layers

J. Shmoys

An interface between air and plasma (regarded as a dielectric with negative dielectric constant) is capable of supporting (E mode) surface waves. Since it is impossible to achieve a sharp boundary between the two regions, it is of interest to investigate the problem of propagation of surface waves when the transition from air to plasma is gradual. The E mode problem for this case of smooth variation is rather difficult; therefore, it was felt that some insight might be obtained from considering semi-infinite regions of air and plasma separated by a homogeneous slab of intermediate density. This problem was solved by the application of the transverse resonance procedure. It turns out that the dispersion curve for the case of an intermediate

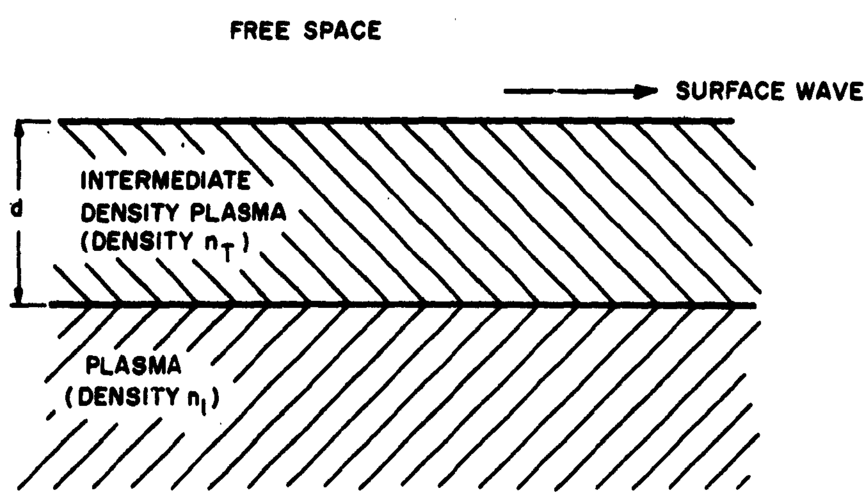

Fig. 13 - Transition layer geometry

density slab differs considerably from that for the abrupt transition. In the latter case, there is a single low frequency pass-band for the surface wave, extending up to $\omega_{p_1}/\sqrt{2}$ (ω_{p_1} is the plasma frequency of the semi-infinite plasma region). The transition layer has the effect of shifting the cut-off frequency of that band downward and of introducing another pass-band at somewhat higher frequencies (but still below ω_p). In addition, if the transition layer is thin, two distinct modes of propagation may exist

near the cut-off frequency of the lower band, one of these being a backward wave. The detailed analysis of the problem and numerical results were issued in an Electrophysics Memorandum, PIBMRI-1093-62.[1] Typical dispersion curves are sketched on Fig. 14.

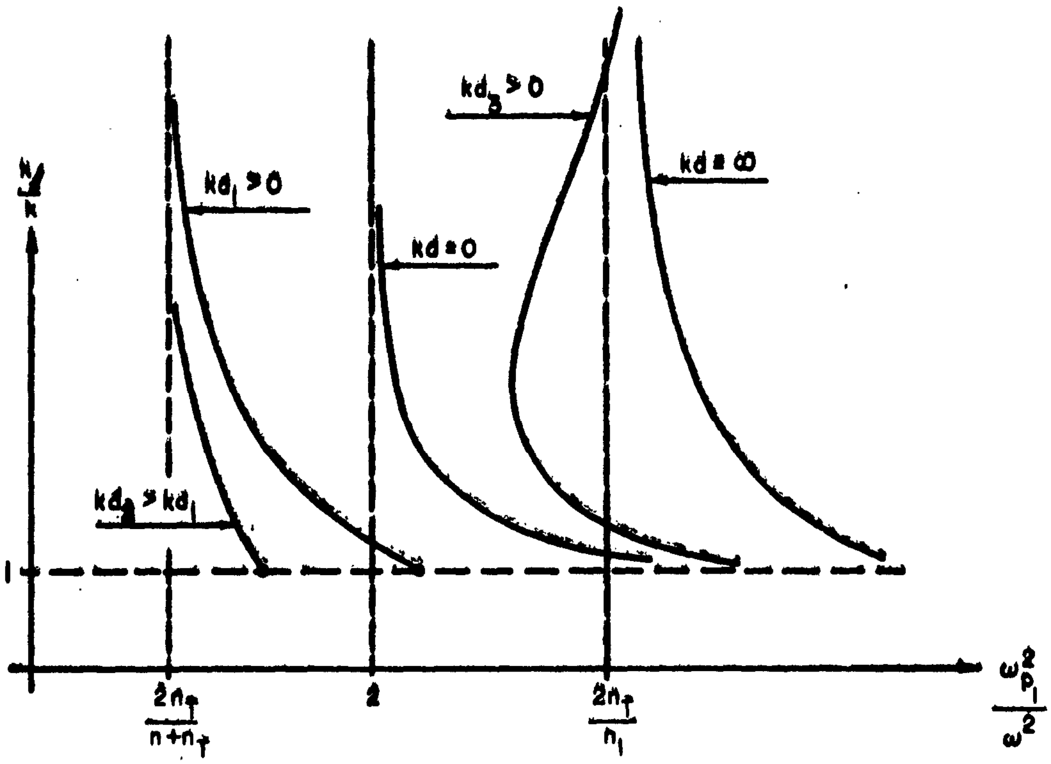

Fig. 14 - Dispersion curves for a transition layer

2. Inhomogeneous media

a. Diffraction by objects in a certain variable plasma medium

A transformation, also mentioned in Section E2, can be employed to relate certain three-dimensional problems of diffraction by rotationally symmetric objects in a homogeneous medium to equivalent two-dimensional diffraction problems in an inhomogeneous medium. The two related configurations are schematized in Fig. 15. The three-dimensional problem in a medium characterized by the constant wave-

1. P. Hirsch and J. Shmoys, "Surface Waves on Plasma Slabs", Electrophysics Memorandum 85, PIBMRI-1093-62, Microw. Res. Inst., Polytech. Inst. of Brooklyn, October 18, 1962.

number k_o involves excitation of the rotationally symmetric scatterer by a ring source, while the two-dimensional problem with medium parameter k(y) has a line source parallel to the obstacle axis. The interesting feature of the latter class of diffraction problems is firstly, that the frequency characteristics of the medium are those of a lossless plasma with a variable electron density and secondly, that the (y) direction along which the medium varies need not bear any relation to the surface of

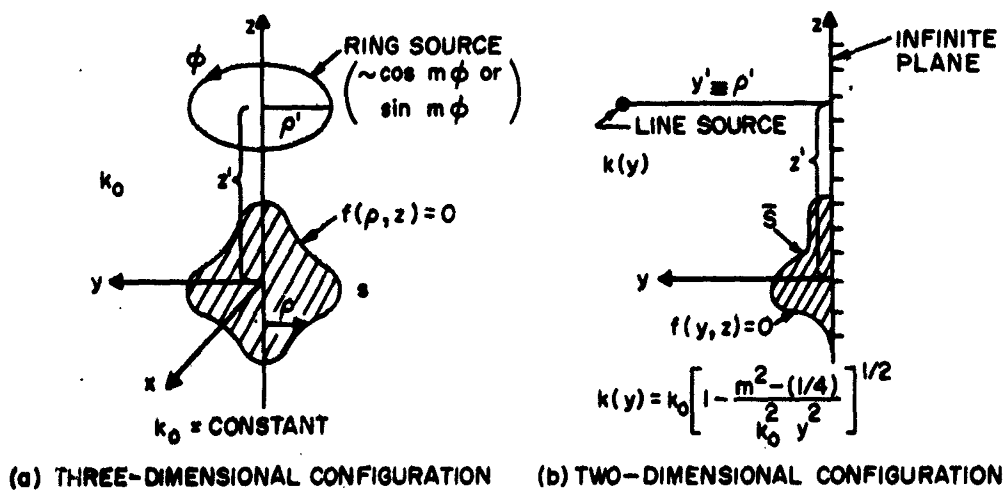

Fig. 15 - Related diffraction problems

the scatterer. Thus, a study of the rigorous solutions for various obstacle configurations permits the investigation of wave perturbations caused by refractive index variations on the scatterer surface, thereby providing a more general class of canonical problems than those available in the literature.

The analysis and some preliminary results have been given in reference 1.

1. L. B. Felsen, "Diffraction by Objects in a Certain Variable Plasma Medium", Electrophys. Group Memo. 55, R-765-59, PIB-693, Microw. Res. Inst., Polytech. Inst. of Brooklyn, July 1959.

b. Determination of plasma density from diffraction measurements*

J. Shmoys

The standard method of microwave probing of plasmas is usually a slight modification of the diagnostic method developed for ionospheric sounding. While in ionospheric sounding the time delay in reflection is measured as a function of frequency, in plasma diagnostics the phase shift in either reflection or transmission is determined. Under some restrictions one can reconstruct the profile of the medium from the variation of these quantities with frequency. A plane stratified medium and an incident plane wave is usually assumed. The two difficulties mentioned above are: (1) frequency must be varied and (2) the plasma must be plane stratified. The variation of frequency over a wide range is an experimental nuisance in the microwave range. A planar configuration in a plasma is sometimes difficult to obtain. Many plasmas which have cylindrical symmetry, would have to be extremely large so that cylindrical stratification could be considered approximately plane.

It is in order to avoid these difficulties that the present diagnostic method is being proposed. We assume a cylindrical geometry and an incident plane wave of fixed frequency. If we now measure the differential scattering cross-section, i.e., the scattered wave amplitude vs. angle, we can reconstruct the density profile. The assumptions are essentially the same as in ionospheric sounding: the variation of electron density with radius must be slow (fractional charge per wavelength must be small) so that the ray theory of propagation of electromagnetic waves be applicable; the density variation must be monotonic; the collision frequency must be sufficiently small so that its effects are negligible. Under these assumptions the calculation of density from the scattering data can be broken up into two parts: the calculation of the ray geometry from the scattering data, and the calculation of the electron density distribution from the ray geometry. The relation between the deflection angle of a ray as a function of distance between that ray and the central ray (impact parameter) in the incident wave (cf. Fig. 16) is obtained from the differential scattering cross-section by a consideration of conservation of energy.

We have to distinguish between the somewhat simpler case of an impenetrable (overdense) plasma in which case the ray deflection angle is a monotonic function of the impact parameter, and the more involved one of a penetrable plasma,

*This topic was mentioned in the previous Final Report dealing with Contract Items 2-4. It is also included here in order to present a complete picture of the variable medium studies carried out under this Contract.

in which case the deflection angle has a maximum. In the latter case, two rays contribute in any given direction and one has to distinguish between the two contributions by making use of the interference pattern. Once the scattering data has been translated into a relationship between scattering angle and impact parameter, one may proceed to calculate the electron density distribution in two different ways.

We can calculate the deflection angle as a function of the impact parameter for an arbitrary radial variation of electron density and obtain an integral expression involving the electron density distribution n which may be regarded as an integral equation for n. This equation is of the Abel type and can be solved. The final result yields a pair of expressions for the electron density and radius in terms of a parameter; these constitute a relationship between density and radius in parametric form. Since these expressions involve the evaluation of integrals of products of scattering angle (available in numerical form from experimental data) and a singular kernel, they are difficult to compute numerically. A class of analytic scattering angle functions was introduced, for which the calculation can be carried out. One can then choose from this class a function that fits the experimental data best and use the corresponding analytical result for the electron density distribution.

Alternatively, one can avoid the solution of the integral equation altogether and simply calculate the scattering angle function for some models of electron density distribution. One then chooses that model for which the calculated scattering angle function fits the experimental data best. Three models were analyzed in this manner.

In all the cases of an underdense (penetrable) column which were considered, a relationship between the maximum scattering angle θ_o and V_a, the ratio of maximum electron density in the column to the density at which the plasma becomes impenetrable, was examined. It was found that in each case this relationship could be reasonably well approximated by

$$V_o = \sin \theta_o \qquad (1)$$

so that maximum plasma density could be deduced from maximum scattering angle.

The details of the above calculations were issued in a report[1] and subsequently published[2].

1. J. Shmoys, "A Proposed Diagnostic Method for Cylindrical Plasmas", PIBMRI-828-60, Microw. Res. Inst., Polytech. Inst. of Brooklyn, May 1960.
2. J. Shmoys, "A Proposed Diagnostic Method for Cylindrical Plasmas", J. Appl. Phys., 32, pp. 689-695, April 1961.

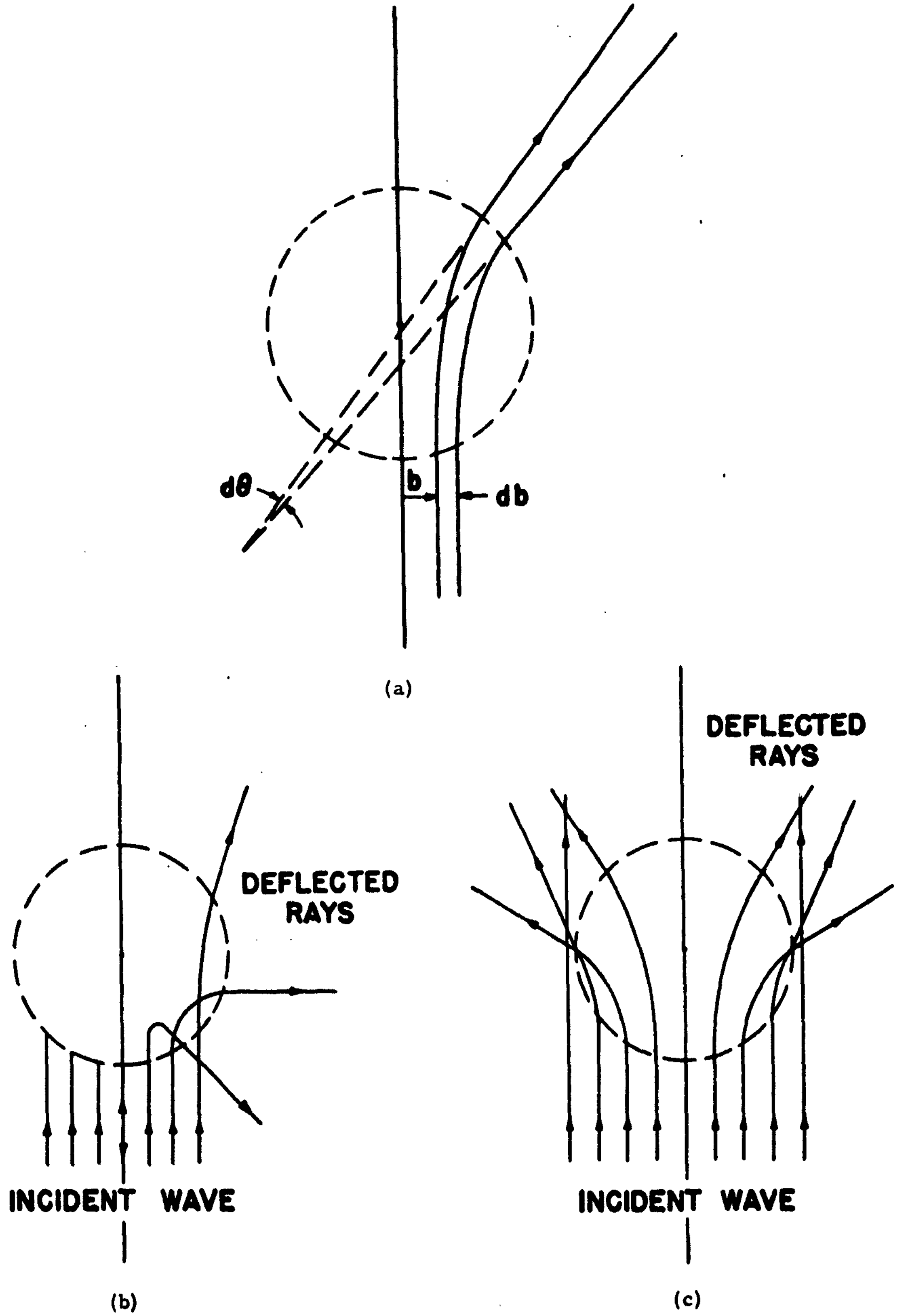

Fig. 16 - Ray paths in cylindrically stratified medium

c. Reflection and transmission of electromagnetic waves in the presence of electron density gradients

J. Shmoys

Albini and Jahn[1] published a calculation of the complex reflection and transmission coefficients for electromagnetic waves incident normally on a plane stratified transition between free space and a lossy isotropic plasma. The variation of electron density [dielectric constant] in the transition region was assumed linear or piecewise linear; the variation of magnitude and phase of the transmission and reflection coefficients was calculated as a function of thickness of the transition layer. Albini and Jahn concluded that the coefficients depend strongly on the width of the transition zone and less strongly on the detailed profile of the transition; they observed periodicity in the amplitude and phase of the reflection coefficient.

In order to check the above conclusions, the variation of the same quantities with effective layer thickness was calculated for the Epstein (inverse hyperbolic tangent) transition[2]. On the basis of the results obtained, it can be said that Albini and Jahn's conclusions are not warranted. The magnitude of the reflection coefficient tends to zero with increasing layer thickness much faster for the Epstein transition, and in a completely different manner. The phase of the reflection coefficient does not show the strong periodic component exhibited in the linear case. It is likely that the behavior of these quantities in the case of a linear or piecewise transition is related to discontinuities in slope.

It was originally planned to issue the results for the Epstein transition in a memorandum but some numerical mistakes were found in the computation of phase and it was felt that all phase curves presented in progress reports[2] should be recalculated; this has not yet been done.

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1. F.A. Albini and R.G. Jahn, "Reflection and Transmission of Electromagnetic Waves at Electron Density Gradients", J. Appl. Phys., 32, pp. 75-82, Jan. 1961.
2. 10th, 11th and 12th Quarterly Reports.

D. Guiding and Scattering Properties of Variable Impedance and Absorbing Surfaces

1. Introduction

A reasonably extensive literature exists on problems of diffraction by perfectly conducting objects of simple shape, describable by single coordinate surfaces in an appropriate coordinate system which renders the vector or scalar wave equation separable. Rigorous field solutions are obtained by the method of separation of variables, and their asymptotic evaluation and subsequent physical interpretation in the limit of high and low frequencies grants considerable insight into the diffraction mechanism in these frequency domains. Keller's geometrical theory of diffraction[1] has been particularly promising in systematizing the various diffraction processes which take place on relatively arbitrary objects in the optical (short-wavelength) limit. The validity of this as yet non-rigorous theory has been verified in special cases by comparing its predictions with results obtained from the above mentioned class of exactly solvable problems. The quasi-optic field comprises the conventional geometric-optical incident and reflected rays, and also diffracted rays which are launched by certain special points on the object surface (e.g., edges, corners, tips, geometrical shadow boundaries, etc.) and which account for diffraction effects. Keller's theory can generally not predict the starting amplitude of a diffracted ray but requires for this information the rigorous solution of a "canonical" problem which is locally similar to a given scattering object.

When applying an asymptotic theory such as Keller's, or its equivalent, a point of importance concerns its limits of applicability. The mathematical requirement $k \to \infty$, where k is the wavenumber in the medium, can frequently be relaxed and phrased as $kd \geq C$, where d is a characteristic dimension of the scatterer and C a bounded positive constant. For example, in the problem of diffraction by a perfectly conducting sphere or cylinder with radius d, the first-order asymptotic results are quite good when $C \approx 10$. To make similar comparisons for non-perfectly conducting objects, it is necessary to obtain rigorous solutions for simple scatterers which exhibit the desired surface impedance properties. Seemingly least complicated is the specification of a constant, non-zero surface impedance Z_s. It is found, however, that many problems which are separable when $Z_s = 0$ cease to be so when Z_s equals a non-vanishing constant. A well-known example in this category is the perfectly conducting wedge or half-plane; paradoxically, the wedge problem (and others) separate for certain special variations of Z_s. Thus, our quantitative knowledge of diffraction processes at high frequencies is much less complete when $Z_s \neq 0$ since exact solutions

are scarcer and more difficult to obtain.

At the time of initiation of this contract, no rigorous results were available for scattering by a variable impedance structure. A non-constant surface impedance is likely to involve limitations additional to those arising from the object's structural shape since the impedance variation per local wavelength introduces a new parameter. While quasi-optic approximations may be expected to hold for slow variations they are bound to be inaccurate or even invalid when the variation is rapid. To confirm these expectations, three types of structures were investigated: a) a wedge and a cone with a surface impedance (or admittance, depending on the polarization of the incident field) which varies linearly with distance from the edge or tip, respectively; b) a circular cylinder with a surface impedance having a sinusoidal peripheral variation about a constant value; and c) a plane surface with a relatively arbitrary (bilinear) variation of Z_s. These choices were dictated primarily by the fact that rigorous results are obtainable for certain two-dimensional forms of excitation: complete solutions in category a) can be found by the method of separation of variables, while those in the non-separable category b) are in the form of a perturbation series. A novel method was applied to solve problem c). The structures in a) permit the study of the effects of impedance variation on plane and quasi-plane surfaces, and near an edge or tip, while those in b) highlight the effects of impedance change on a curved object. Item c) generalizes some of the results in a) by permitting a more general type of impedance variation. From an investigation of the asymptotic properties of these solutions, carried out for a) and b), estimates on the limits of validity of geometrical optics were obtained, and results were found for the field behavior when geometrical optics is no longer applicable. During the study of the problems in a), a new type of surface wave was discovered which propagates along a variable impedance plane, and which is of special interest in connection with radiation from a tapered surface wave antenna.

To assess the influence of a constant surface impedance on the field diffracted by an edge, a study of diffraction by an imperfectly conducting half-plane was also carried out, with emphasis on those values of Z_s which yield maximum absorption of the incident wave (black screen).

2. Wedge or cone with a linearly varying surface impedance

This study deals with a line-source excited wedge (Fig. 17(a)) or a ring-source excited cone (Fig. 17(b)) whose surface impedance (or admittance) varies linearly with distance from the apex or tip, respectively. For the wedge problem, two

different impedance values Z_{so} and $Z_{s\alpha}$ are permitted on the faces at $\phi = 0$ and $\phi = \alpha$, respectively; when the exciting line source comprises magnetic currents, one has $Z_{so,\alpha} = ik\rho c_{o,\alpha}$, where c_o and c_α are constants subject to the restriction $\mathrm{Im}\, c_{o,\alpha} \leq 0$ to assure passivity. k is the real wavenumber in the surrounding medium and ρ is the radial variable. Analogous considerations apply to Z_s in the cone problem except that ρ is replaced by the spherical variable r. These boundary value problems can be solved by the method of separation of variables, and alternative representations of the solution were constructed which are especially suited to the study of the various

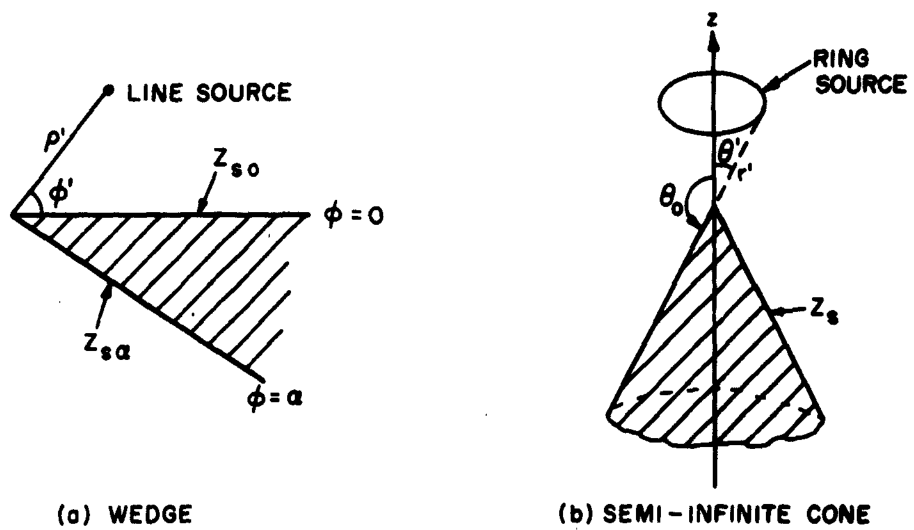

Fig. 17 - Physical structures

propagation and diffraction phenomena.

a. Formulation emphasizing surface guiding properties

If the wedge in Fig.17(a) is viewed as a radial waveguide in which cylindrically spreading or contracting waves propagate, the appropriate representation involves the complete set of (discrete) angular eigenfunctions. During the investigation of the angular eigenvalue problem, a new type of surface wave was discovered[2] which is guided by one or both of the wedge faces, propagates outward from the apex like a cylindrical wave, and decays in the ϕ-direction away from the wedge surface. Its energy in the region between the wedge face at $\phi = 0$ and a plane parallel to $\phi = 0$, for example, does not remain constant; instead, continuous radiation takes place because

of the surface impedance variation. This wave is particularly interesting since it may serve as a rigorous prototype for the analysis of a tapered surface wave antenna. A theoretical study of the latter structure was carried out[2] and it was shown that very high endfire gains can be achieved if a constant reactance surface waveguide is terminated by a suitable tapered reactance section from which the radiation takes place. The calculations also included data on the sidelobes to be expected from the junction between the two waveguide sections.

The field representation in terms of radially propagating waves is also useful for the study of the diffraction fields near the apex. Analogous considerations apply to the cone problem.

b. Formulation emphasizing scattering properties

To analyze the scattering properties of the wedge (or cone) when both the source and observation points are located many wavelengths from the apex, an angular transmission representation is appropriate. From this viewpoint, waves are regarded to propagate along the ϕ (or θ) directions, and the field representation is in terms of the (continuous) set of radial eigenfunctions. In its most elementary form, the angular space is infinitely extended, and the effects of boundaries delimiting the finite physical ϕ (or θ) space are taken into account by multiple reflection or image construction. The general utility of the angular transmission approach in the analysis of high-frequency diffraction problems was discussed in reference 3. A first-order asymptotic evaluation of the formal integral solution yields a result for the far field which can be interpreted conveniently in terms of the geometric-optical ray picture shown in Fig. 18.

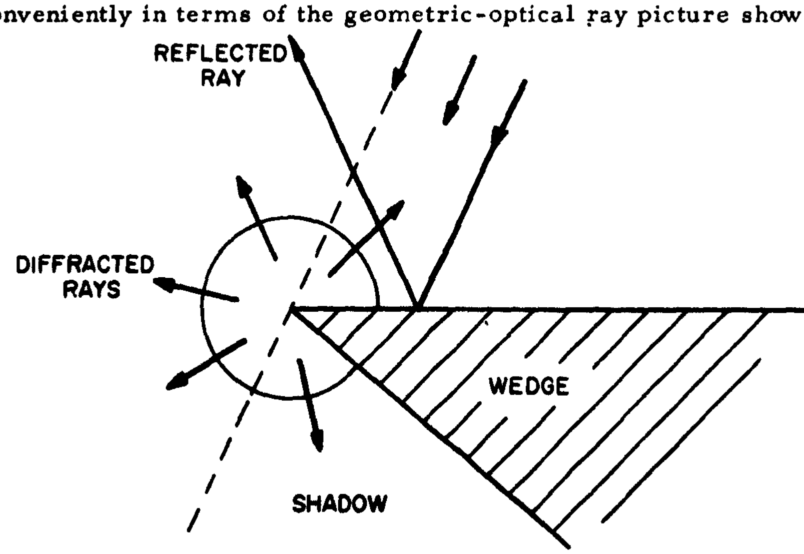

Fig. 18 - Quasi-optic fields

The field comprises direct and reflected rays - each in a domain predicted from geometrical optics; in addition, a cylindrical diffracted wave emanating from the edge is observable in both the illuminated and shadow regions. The reflected ray amplitude is determined by the local wedge impedance at the point of reflection, thereby confirming the validity of its construction by geometric-optical considerations. The diffracted wave amplitude D is found, however, to depend not only on the impedance at the diffraction point (the edge) but is influenced by its rate of variation in the vicinity of the edge. If the surface impedance variation at $\phi = 0$ is written as $A\rho$, and the wedge face at $\phi = \alpha$ is assumed to be perfectly conducting, then when $A \ll 1$, D is essentially the same as for a zero impedance wedge; however, when $A \gg 1$, D is essentially the same as for a wedge with infinite impedance at $\phi = 0$ although the impedance at the edge $\rho = 0$ is zero. $A \ll 1$ and $A \gg 1$ correspond, respectively, to slow and rapid impedance variations near the edge whence it is noted that the geometric-optical predictions apply approximately for slow, but are invalid for rapid, variation. The behavior of D for intermediate ranges of A has been calculated in reference 2 which also contains a complete asymptotic expansion of the diffraction field.

3. Circular cylinder with peripherally varying surface impedance

C. J. Marcinkowski and L. B. Felsen

This section deals with the diffraction of a line source current field by a cylinder with the sinusoidally varying surface impedance [3]

$$Z_s(\phi) \to \sqrt{\frac{\mu}{\epsilon}} \left[1 + \alpha \cos p(\phi - \phi_o)\right] \tag{1}$$

where ϕ is the angle of observation, ϕ_o is an arbitrary phase angle, and p expresses the periodicity of the surface impedance. This study was undertaken for the purpose of exploring the simultaneous influence of surface curvature and variability of surface properties on the field diffracted by an object at high frequencies. The sinusoidal impedance variation is highly desirable since an appropriate choice of the periodicity parameter p permits the simulation of slow, monotonic variations on the one hand, and of rapid fluctuations on the other. For $|\alpha| \ll 1$, the problem can be solved by a perturbation method which leads to a Green's function representation patterned on the well known results for a constant impedance cylinder. [3] The infinite angular transmission line approach mentioned in Sec. 2b was also employed here to derive solutions for the constant impedance cylinder which permit a direct asymptotic field evaluation in both the illuminated and shadow regions. These results were then extended to include the variable impedance case for which the fields were expressed as power series

involving the perturbation parameter α.

These formal solutions were then investigated to first order in α for a plane wave incident on a large cylinder.[3] A saddle point evaluation yielded a result which is interpretable in terms of the geometrical optics appropriate to a cylindrically curved, convex reflection grating. The solution comprises the first and higher order reflected rays familiar from the theory of the plane grating, modified by appropriate divergence coefficients.[3] This extension of conventional geometrical optics provides a complete and quantitative physical picture of the mechanism of reflection of waves from a curved surface with periodic impedance variation.

For a slowly varying surface impedance ($p \sim 1$), conditions were derived under which the various reflected rays in the solution to order α may be combined into a single, specularly reflected ray with a reflection coefficient depending only on the local impedance value at the point of reflection.[3] Since this is precisely the behavior of the geometric-optical field reflected from a variable, non-periodic impedance surface these conditions which involve both the rate of change of surface impedance and the radius of curvature of the surface, serve as criteria for the validity of geometrical optics when applied to curved, variable impedance structures.

4. Plane surface with bilinear impedance variation

C. J. Marcinkowski

The problem of diffraction by a linearly varying surface impedance on a wedge and cone was solved previously by the method of separation of variables (see Sec. 2). To deal with the problem of a linearly varying impedance on an infinite plane surface, separation of variables is inapplicable and other methods of analysis must be employed. The impedance function is assumed to have the form

$$Z(x) = \sqrt{\frac{\mu}{\epsilon}}\,(\zeta_o + \zeta_1 kx) \quad , \qquad -\infty < x < \infty \quad , \tag{2}$$

where k is the free-space wavenumber, $\sqrt{\mu/\epsilon}$ the free-space impedance, and ζ_o and ζ_1 are arbitrary complex numbers. By a combination of Fourier integral and integration-by-parts procedures, the problem for the unknown Fourier transform is reduced to a first-order differential equation which was solved explicitly by a standard procedure.[4] Since this method of solution has apparently not been considered before, an exploration of this technique was carried out to determine the extent to which it could be developed. This led to the discovery of the formal solutions for a surface impedance in the bilinear form[5]

$$Z(x) = \sqrt{\frac{\mu}{\epsilon}}\ \frac{\zeta_o + \zeta_1 kx}{a_o + a_1 kx} \qquad (3)$$

where a_o and a_1 are arbitrary complex numbers.

The solution for the linear impedance (2) in the neighborhood $|\zeta_1| << 1$ is of physical importance since it involves the interesting quasi-optical region of a slowly varying surface impedance.[6, 7] This neighborhood is also of mathematical importance because the solution contains the factor $1/\zeta_1$ in an exponential and as a multiplier in front of the exponential,[10] thereby exhibiting an apparent singularity at $\zeta_1 = 0$. For these reasons the limiting behavior $|\zeta_1| \to 0$ was investigated for the physically important case of a reactive surface impedance. It was found[7] that solutions were allowed for a constant inductive reactance $\arg(\zeta_o) \simeq -\pi/2$, and for either type of varying reactance with $\arg(\zeta_1) \simeq \pm\pi/2$. Dual results were obtained for surface admittance functions. This produced the formal solutions for the infinite plane with the bilinearly varying surface impedance[5] but no time was available for a detailed examination of the physical properties of any of these solutions.

5. Diffraction by an absorbing half-plane

C. J. Marcinkowski

Although the two-dimensional electromagnetic problem of diffraction by a constant impedance half-plane has been studied by previous workers, there still remain two major deficiencies in the studies reported in the literature. One of these lies in the mathematical complexity of the formal solution while the other is due to the very narrow range of surface impedance values which have been examined. Several steps were taken to simplify the solution. The first was to make use of a much simpler but hitherto neglected result obtained by Grinberg and Fock[8] and used extensively by the Russian school of investigators. The surface impedance was described in terms of an angle of "perfect absorption" designated as ϕ_a, the angle at which an incident plane wave produces no reflected plane wave. By means of this procedure, the asymptotic far-field scattering patterns could be specified in a physically significant manner in terms of three similar, dimensionless parameters[8]: the angle of incidence ϕ_i, the angle of observation ϕ, and the angle of perfect absorption ϕ_a.

Since previous investigations have been restricted to the narrow range of small surface impedances, they have supplied little information concerning the influence of surface impedance on the scattered far fields. In contrast, the scattered far fields were studied under this Contract for a relatively arbitrary, constant surface

impedance which may act like a black screen.[8] A null was discovered in the back-scattered fields with properties which permitted the interpretation of the asymmetric patterns of the constant impedance half-plane in terms of an angular deformation or distortion of the more symmetric patterns of a perfectly reflecting structure.[8] The scattering patterns were discovered to be roughly similar to those of the Sommerfeld black screen[8] which has the very simple amplitude behavior $1/\left[\pi^2 - (\pi + \phi_i - \phi)^2\right]$. All these simplifications permit us to sketch out the main properties of the diffracted far fields in a simple qualitative manner without the need for elaborate calculations.[8]

References

1. J. B. Keller, "Geometrical Theory of Diffraction", J. Opt. Soc., Vol. 52, 1962, pp. 116-130.

2. L. B. Felsen, "On the Electromagnetic Properties of Wedge and Cone Surfaces with a Linearly Varying Surface Impedance", Report R-736-59, Microw. Res. Inst., Polytech. Inst. of Brooklyn, April 1960.

3. C. J. Marcinkowski and L. B. Felsen, "Diffraction by a Cylinder with a Variable Surface Impedance", Report PIBMRI-821-60, Dept. of Electrophysics, Polytech. Inst. of Brooklyn, Jan. 1961.

4. 13th Quarterly Report on this Contract, December 18, 1961.

5. 14th Quarterly Report on this Contract, February 19, 1962.

6. 15th Quarterly Report on this Contract, May 11, 1962.

7. 16th Quarterly Report on this Contract, July 26, 1962.

8. C. J. Marcinkowski, "Diffraction by an Absorbing Half-Plane", Report R-750-59, PIB-678, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 1959.

E. Other Diffraction Problems

1. Plane wave back-scattering from a semi-infinite cone

In previous studies of diffraction by a semi-infinite cone[1], it has been shown that the far field can be decomposed into incident, reflected and diffracted constituents. The first two are the geometric-optical field contributions which can be evaluated in closed form while the third involves a canonical diffraction integral which specifies the angular variation of the spherical wave emanating from the cone tip. Since an exact evaluation of the diffraction integral does not seem possible, it was decided to carry out an approximate evaluation for the special case of plane wave back-scattering along the cone axis. Approximate results had previously been obtained for the limiting cases of cones having very small or large apex angles[2]; the present study yielded a formula applicable for arbitrary angles, and the resulting magnitude of the back-scattered field has been given in the 1st Quarterly Report (see also reference 3).

2. Relation between a class of two-dimensional and three-dimensional diffraction problems

By means of a certain transformation, a relationship was demonstrated between a class of two-dimensional and three-dimensional scalar or electromagnetic diffraction problems. The basic three-dimensional configuration consists of a perfectly reflecting half-plane excited by a ring source centered about the edge and having a variation $\exp(\pm i\phi/2)$, where ϕ is the azimuthal variable; in addition, a perfectly reflecting rotationally symmetric obstacle whose surface is defined by $f(\rho, z) = 0$ (ρ, z are cylindrical coordinates), may be superposed about the edge (z-axis). This problem was shown to be simply related to the two-dimensional one for the line source excited configuration $f(y, z) = 0$, where y and z are Cartesian coordinates (see Fig. 19). Various special obstacle configurations were treated in detail.

For the general case of arbitrary electromagnetic excitation, the above-mentioned transformation was used to construct the solution for the diffraction by a perfectly conducting half-plane from the knowledge of appropriate scalar solutions,

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1. L. B. Felsen, IRE Trans. PGAP, AP-5 (1957).
2. L. B. Felsen, J. A. P., 26 (1955).
3. L. B. Felsen, "Back-Scattering by a Semi-Infinite Cone", Electrophys. Group Memo. 43, R-675-58, PIB-603, Microw. Res. Inst., Polytech. Inst. of Brooklyn, July 1958.

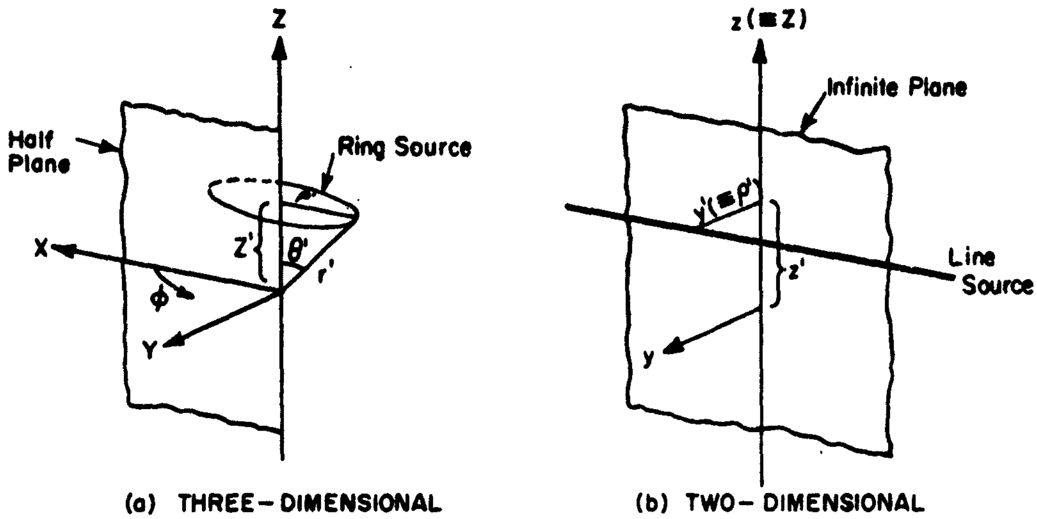

Fig. 19 - Related configurations

namely those which obey the same equations and boundary conditions, and have the same excitations, as the Cartesian components of the electromagnetic field.

The results of this analysis have been reported in reference 1.

F. Cerenkov Radiation

L. B. Felsen and A. Hessel

Cerenkov radiation problems, dealing with radiation from a charge (or charges) in straight uniform motion near or in perturbing media which can support electromagnetic waves with phase speeds slower than the particle speed, have been analyzed in the literature by various methods which become quite involved when the fields are not derivable from a single scalar potential function. For example, a charge moving parallel to the plane boundary of a plane stratified dielectric medium, a plane grating, a plane reactive surface, etc., excites both E and H modes in the z-direction perpendicular to the interface and the associated field problem is a vector problem. While modal network procedures have been applied for some time, and with considerable success, to the analysis of radiation from stationary sources, they had not been utilized for fields excited by moving sources. Such a study was initiated under this contract

1. L. B. Felsen and S. N. Karp, "Relation between a Class of Two-Dimensional and Three-Dimensional Diffraction Problems", Report R-694-58, PIB-622, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Jan. 1959.

(see 9th Quarterly Report), and it was shown how the solution of Cerenkov radiation problems can be systematized through the use of modal techniques by which one decomposes the temporal Fourier transform of the particle field into E and H modes along z. The z-dependent modal amplitudes are evaluated from a network problem wherein the perturbing interface or structure is represented by an equivalent network which generally couples the various modes, and the time dependent fields are recovered by Fourier synthesis.

For example, if a charge moves parallel to a periodic structure (e.g., a grating) (see Fig. 20(a)), the associated network problem is the one schematized in Fig. 20(b). The temporal Fourier transform of the current excitation descriptive of the moving charge is a line source with a linearly progressing phase. This source drives the ($n = 0$, non-propagating) E and H mode transmission lines, and coupling to the propagating mode lines denoted by subscripts n to n+N is achieved by the coupling network representative of the grating. Since the source moves in vacuum where the particle speed is less than the velocity of light, the associated fields are exponentially decaying and radiation occurs only because of the coupling to one or more of the propagating grating modes.

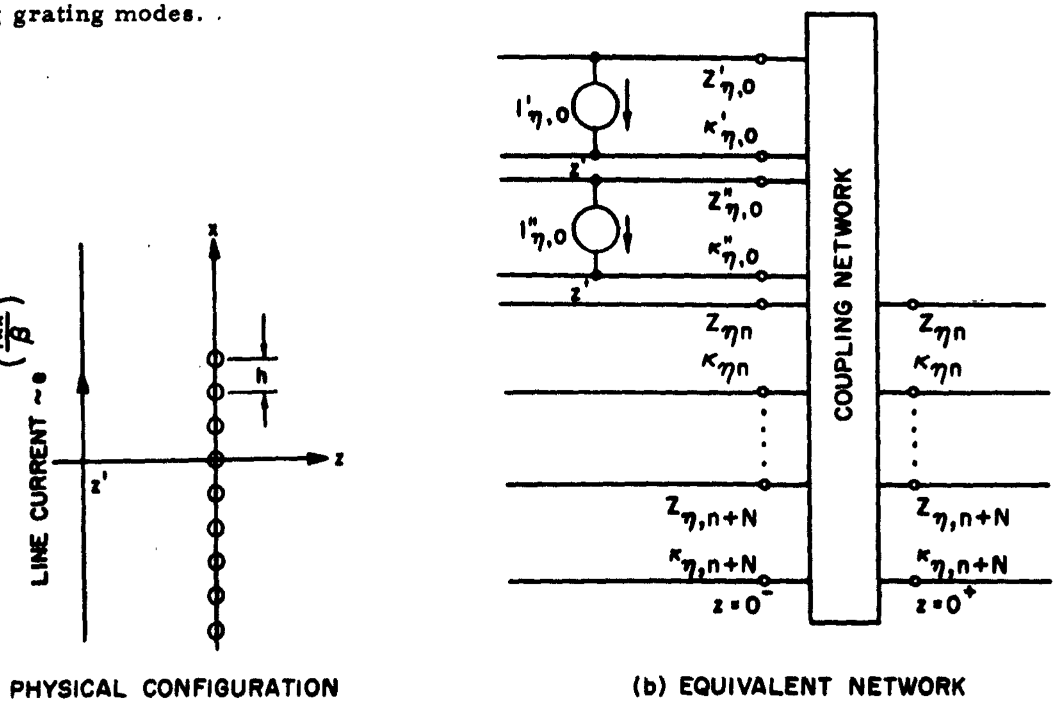

Fig. 20 - Cerenkov radiation

The above-described method has been presented in reference 1, and various applications have been under study. The results will be reported under the successor contract.

G. Network Theory

D. C. Youla

The amount of material reported in this period is varied and quite extensive.

In the first quarterly, July 1958 - September 1958, appeared two notes dealing with parametrically excited structures. The first, "Response of Parametrically Excited Networks", by L. Smilen and D. C. Youla, developed a single-frequency network formalism enabling the analysis of cascades of passive networks with embedded varactors to be carried through in relatively simple fashion. The second, "A Generalization of the Manley-Rowe Relations", by D. C. Youla, developed power frequency formulas for non-hysteretic lossy varactors. The main conclusion was the fact that formulas, unlike the lossless case, depended strongly on the varactor characteristic.

In the second quarterly, October 1958 - December 1958, a note by D. C. Youla entitled "Synthesis of Smooth Non-Uniform Transmission lines from a Prescribed Scattering Coefficient", presented an exact technique for the synthesis of a smooth, lossless non-uniform transmission line (of finite length) from prescribed input reflection datum. The method reduced the problem to the solution of a one-parameter family of Fredholm integral equations. As yet, a truly significant engineering adaptation is not available and a good deal remains to be done along these lines.

In the third quarterly, January 1959 - March 1959, the article, "A Note on the Stability of Linear, Non-Reciprocal N-Ports", by D. C. Youla, suggested a method for determining the real-frequency stability of a non-reciprocal linear, time-invariant n-port via its impedance matrix. Inherent limitations as well as advantages were pointed out. An application to the 2-port case immediately yielded the classical formulas due to Raisbeck.

The fourth quarterly, April 1959 - June 1959, contains, in the paper, "Weissfloch Equivalents for Lossless 2n-Ports" by D. C. Youla, a rather significant contribution to the representation of lossless multi-mode discontinuities. Work on

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1. L. B. Felsen and A. Hessel, "A Network Approach to the Analysis of Cerenkov Radiation Problems", Electrophysics Memorandum No. 60, PIBMRI-878-60, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Nov. 9, 1960.

associated measurement techniques is now in progress.

In "Synthesis of Networks Containing both Positive and Negative Resistors", by D. C. Youla, fifth quarterly, July 1959 - September 1959, a network theory of the usual lumped elements plus negative resistors is initiated. It is shown that many of the passive network techniques and theorems may be used to derive results for the active case. Two specific theorems concerning the number of positive and negative resistors required for the synthesis of lumped active n-ports are derived and their implications discussed in some detail.

In the sixth quarterly, October 1959 - December 1959, a rather complete analysis was presented of the single-frequency invariantive properties of a linear, time-invariant n-port. In the note, "The Analytic Foundations of the "Spot" Frequency Theory of Linear, Noiseless, Time-Invariant n-Ports", by D. C. Youla, the concept of the cross-ratio matrix of four n-ports was introduced for the first time. This notion led naturally to the "self" cross-ratio matrix of a single non-reciprocal n-port. A most important conclusion was that the eigenvalues of this "self" cross-ratio matrix remained invariant under lossless, reciprocal 2n-port embedding. That these eigenvalues reflected everything of physical interest was demonstrated by exhibiting several canonical forms.

The note, "The Inverse Problem for Smooth, Lossless Non-Uniform Transmission Lines", by D. C. Youla, seventh quarterly, January 1960 - March 1960, continues the related research reported in the second quarterly by the same author. Additional theorems and refinements are supplied but the goal of a practical, exact engineering synthesis technique is not attained and further work is necessary.

Many problems in electrical engineering, such as the synthesis of linear n-ports and the detection and filtration of multivariable systems corrupted by stationary additive noise depend for their successful solution upon the factorization of a matrix-valued function of a complex variable p. The paper, "On the Factorization of Rational Matrices", by D. C. Youla, eighth quarterly, April 1960 - June 1960, presents several algorithms for affecting such decompositions for the class of rational matrices G(p); i. e., matrices whose entries are ratios of polynomials in p. The methods employed are elementary in nature and center around the Smith canonical form of a polynomial matrix. Several nontrivial examples are worked out in detail to illustrate the theory.

In "Some Results in the Single-Frequency Theory of Linear Noiseless Networks", by D. C. Youla, ninth quarterly, July 1960 - September 1960, the work initiated in the sixth quarterly is generalized and enlarged. The "Q" matrix of an n-port is introduced and shown to always exist irrespective of the activity character of the structure. The requirements of reciprocity, passivity, losslessness and symmetry are formulated in terms of Q. All cross-ratio matrices are expressed in terms of Q which emerges as an invariantive representation of the n-port. Its transformation properties under 2n-port embedding are remarkably simple and completely avoid the matrix bilinear operators characteristic of the immittance and scattering bases.

In the tenth quarterly, October 1960 - December 1960, a major breakthrough in 2-port insertion-loss filter design is reported in "Darlington Synthesis via Richard's Theorem", by D. C. Youla. Explicit formulas for the element values of the types A, B, C and D sections are derived and summarized in chart form in "A New Theory of Cascade Synthesis", by D. C. Youla, Research Report No. PIBMRI-916-61, May 29, 1961. By means of the chart, the design of a filter may be accomplished with a minimum of numerical labor. In addition, a new theorem on positive-real functions is presented which serves as the basis for a new technique of 1-port cascade synthesis.

Reports and memoranda published under the contract:

E. Arbel, "Radiation from a Point Source in an Anisotropic Medium", PIBMRI-861-60, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Nov. 2, 1960

E. Arbel and L. B. Felsen, "On Electromagnetic Green's Functions for Uniaxially Anisotropic Regions", PIBMRI-985-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Feb. 26, 1962

E. Arbel and L. B. Felsen, "On the Theory of Radiation from Sources in Plane Stratified Anisotropic Plasma Media", PIBMRI-1069-62, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 25, 1962

L. B. Felsen and S. N. Karp, "Relation Between a Class of Two-Dimensional and Three-Dimensional Diffraction Problems", Report R-694-58, PIB 622, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Jan. 12, 1959

L. B. Felsen, "On the Electromagnetic Properties of Wedges and Cones with a Linearly Varying Surface Impedance", Report R-736-59, PIB 664, Microw. Res. Inst., Polytech. Inst. of Brooklyn, April 14, 1960

L. B. Felsen, "Diffraction by Objects in a Certain Variable Plasma Medium", Report R-765-59, PIB 693, (Electrophysics Memo. No. 55), Microw. Res. Inst., Polytech. Inst. of Brooklyn, July 20, 1959

L. B. Felsen and A. Hessel, "A Network Approach to the Analysis of Cerenkov Radiation Problems", PIBMRI-878-60, (Electrophysics Memo. No. 60), Microw. Res. Inst., Polytech. Inst. of Brooklyn, Nov. 9, 1960

L. B. Felsen, "Radiation from a Uniaxially Anisotropic Plasma Half-Space", PIBMRI-1058-62, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Aug. 27, 1962

L. B. Felsen, "On Diffraction by Objects in a Uniaxially Anisotropic Medium", PIBMRI-1073-62, (Electrophysics Memo. No. 83), Microw. Res. Inst., Polytech. Inst. of Brooklyn, Aug. 31, 1962

A. Hessel, N. Marcuvitz and J. Shmoys, "Scattering and Guided Waves at an Interface Between Air and a Compressible Plasma", PIBMRI-953-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 29, 1961

C. J. Marcinkowski, "Diffraction by an Absorbing Half-Plane", Report R-750-59, PIB 678, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 29, 1959

C. J. Marcinkowski and L. B. Felsen, "Diffraction by a Cylinder with a Variable Surface Impedance", PIBMRI-821-60, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Jan. 27, 1961

A. A. Oliner and T. Tamir, "Backward Waves on Isotropic Plasma Slabs", PIBMRI-930-61, (Electrophysics Memo. No. 66), Microw. Res. Inst., Polytech. Inst. of Brooklyn, July 5, 1961

A. A. Oliner and T. Tamir, "Radiation from Semi-Infinite Slot-Excited Plasma Sheath Configurations", PIBMRI-1029-62, Microw. Res. Inst., Polytech. Inst. of Brooklyn, May 17, 1962

Reports and memoranda (continued)

J. Shmoys, "A Proposed Diagnostic Method for Cylindrical Plasmas", PIBMRI-828-60, Microw. Res. Inst., Polytech. Inst. of Brooklyn, May 16, 1960

J. Shmoys and P. Hirsch, "Surface Waves on Plasma Slabs", PIBMRI-1093-62, (Electrophysics Memo. No. 85), Microw. Res. Inst., Polytech. Res. Inst., Oct. 18, 1962

T. Tamir, "The Electromagnetic Field of a Source-Excited Plasma Slab", PIBMRI-932-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 1961

T. Tamir and A. A. Oliner, "The Influence of Complex Waves on the Radiation Field of a Slot-Excited Plasma Layer", PIBMRI-959-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Nov. 2, 1961

T. Tamir and A. A. Oliner, "The Spectrum of Electromagnetic Waves Guided by a Plasma Layer", PIBMRI-970-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Dec. 13, 1961

D. Youla, "A Generalization of the Manley-Rowe Relations", R-702-58, PIB 631, (Networks and Waveguide Memo. No. 16), Microw. Res. Inst., Polytech. Inst. of Brooklyn, Dec. 17, 1958

D. Youla, "Single Frequency Synthesis from a Prescribed Scattering Matrix", R-721-59, PIB 649, (Networks and Waveguide Memo. No. 20), Microw. Res. Inst., Polytech. Inst. of Brooklyn, March 15, 1959

D. Youla, "Weissfloch Equivalents for Lossless 2 N-Ports", R-753-59, PIB 681, (Networks and Waveguide Memo. No. 25), Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 4, 1959

D. Youla, "Some New and Useful Matrix Results", PIBMRI-807-60, (Networks and Waveguide Memo. No. 34), Microw. Res. Inst., Polytech. Inst. of Brooklyn, March 7, 1960

D. Youla, "On the Factorization of Rational Matrices", PIBMRI-855-60, Microw. Res. Inst., Polytech. Inst. of Brooklyn, Sept. 20, 1960

D. Youla, "A Normal Form for a Matrix under the Unitary Congruence Group", PIBMRI-858-60, (Networks and Waveguide Memo. No. 41), Microw. Res. Inst., Polytech. Inst. of Brooklyn, Aug. 30, 1960

D. Youla, "Darlington Synthesis via Richards Theorem", PIBMRI-901-61, (Networks and Waveguide Memo. No. 50), Microw. Res. Inst., Polytech. Inst. of Brooklyn, March 14, 1961

D. Youla, "A New Theory of Cascade Synthesis", PIBMRI-916-61, Microw. Res. Inst., Polytech. Inst. of Brooklyn, May 29, 1961

D. Youla, "Cascade Synthesis", PIBMRI-992-62, (Networks and Waveguide Memo. No. 62), Microw. Res. Inst., Polytech. Inst. of Brooklyn, Feb. 14, 1962

D. Youla, L. Kaplan and D. Stock, "Analytical Foundations of Linear Time-Invariant N-Ports", PIBMRI-1028-62, Microw. Res. Inst., Polytech. Inst. of Brooklyn, May 3, 1962

Reports and memoranda (continued)

D. Youla, "Limits of Errors in Mismatched Attenuators, Part I", PIBMRI-1042-62, (Networks and Waveguide Memo. No. 68), Microw. Res. Inst., Polytech. Inst. of Brooklyn, June 19, 1962

Papers published in scientific journals:

E. Arbel and L. B. Felsen, "On the Theory of Radiation from Sources in Anisotropic Plasmas, Parts I and II", to be published in the Proceedings of the 1962 Copenhagen Symposium on Electromagnetic Theory and Antennas

L. B. Felsen, "Electromagnetic Properties of Wedge and Cone Surfaces with a Linearly Varying Surface Impedance", IRE Trans., AP-7, 231 (1959)

L. B. Felsen and S. N. Karp, "Relation Between a Class of Two-Dimensional and Three-Dimensional Diffraction Problems", IRE Trans., AP-8, 407 (1960)

L. B. Felsen, "Radiation from a Tapered Surface Wave Antenna", IRE Trans. PGAP, (1960)

L. B. Felsen and A. Hessel, "A Network Approach to the Analysis of Cerenkov Radiation Problems", Nuovo Cim. - Serie X, 19, 1065, (1961)

L. B. Felsen, "High-Frequency Diffraction by a Wedge with Linearly Varying Surface Impedance", Appl. Sci. Res., B9, 170 (1961)

L. B. Felsen, "Lateral Waves on an Anisotropic Plasma Interface", IRE Trans. PGAP (Communications), AP-10, No. 3, (1962)

L. B. Felsen, "Radiation from a Uniaxially Anisotropic Plasma Half-Space", to be published in IEEE Trans. PGAP

A. Hessel, N. Marcuvitz and J. Shmoys, "Scattering and Guided Waves at an Interface Between Air and Compressible Plasma", IRE Trans. PGAP, AP-10, 48 - 54, Jan. 1962

C. J. Marcinkowski, "Some Diffraction Patterns of an Absorbing Half-Plane", Appl. Sci. Res., B9, 189-198 (1961)

C. J. Marcinkowski and L. B. Felsen, "On High-Frequency Diffraction by Curved Surfaces with Periodic Impedance Properties", J. Res. NBS, 66D, No. 6, 699 (1962)

L. B. Felsen and C. J. Marcinkowski, "Diffraction by a Cylinder with a Variable Surface Impedance", Proc. Roy. Soc. London, A267, 329-350 (1962)

C. J. Marcinkowski and L. B. Felsen, "On the Geometrical Optics of Curved Surfaces with Periodic Impedance Properties", J. Res. NBS, 66D, 699-705 (Nov. - Dec. 1962)

C. J. Marcinkowski and L. B. Felsen, "On the Limitations of Geometrical Optics Solutions for Curved Surfaces with Variable Impedance Properties", J. Res. NBS, 66D, No. 6, 707-712 (Nov. - Dec. 1962)

Papers published in scientific journals (continued)

A. A. Oliner and T. Tamir, "Backward Waves on Isotropic Plasma Slabs", J. Appl. Phys., 33, 231-233, Jan. 1962

A. A. Oliner and T. Tamir, "Radiation from Semi-Infinite Slot-Excited Plasma Sheath Configurations", Second Symposium on the Plasma Sheath, Plenum Press, to be published

J. Shmoys, "Diffraction by a Half-Plane with a Special Impedance Variation", IRE Trans. PGAP, AP-7, S88-S90, Dec. 1959

J. Shmoys and E. Mishkin, "Hydromagnetic Waveguides with Finite Conductivity and Arbitrary Cross-Section", Phys. and Fluids, 3, 473 (1960)

J. Shmoys, "A Proposed Diagnostic Method for Cylindrical Plasmas", J. Appl. Phys., 32, 689 (1961)

T. Tamir and A. A. Oliner, "The Influence of Complex Waves on the Radiation Field of a Slot-Excited Plasma Layer", IRE Trans. PGAP, AP-10, 55-65, Jan. 1962

T. Tamir and A. A. Oliner, "The Spectrum of Electromagnetic Waves Guided by a Plasma Layer", Proc. IEEE, Vol. 51, 317-332, Feb. 1963

D. Youla, "A Normal Form for a Matrix under the Unitary Congruence Group", Can. Jour. Math., Vol. 13, 694-704, 1961

D. Youla, "On the Factorization of Rational Matrices", IRE Trans. on Information Theory, July 1961

D. Youla, "New Theory of Cascade Synthesis", IRE Trans. on Circuit Theory, Sept. 1961

Talks presented at technical meetings:

E. Arbel and L. B. Felsen, "Radiation from Sources in Anisotropic Plasmas", Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962

L. B. Felsen, "Electromagnetic Properties of Wedge and Cone Surfaces with a Linearly Varying Surface Impedance", Symposium on Electromagnetic Theory, Univ. of Toronto, July 1959

L. B. Felsen, "Diffraction by Objects in a Certain Variable Medium", URSI Spring Meeting, Washington, D. C., 1960

L. B. Felsen and C. J. Marcinkowski, "Diffraction by a Cylinder with a Variable Surface Impedance", URSI Spring Meeting, Washington, D. C., 1960

L. B. Felsen, "Lateral Waves on an Anisotropic Plasma Interface", URSI Spring Meeting, Washington, D. C., 1962

Talks presented at technical meetings (continued)

L. B. Felsen, "Two-Dimensional Radiation and Diffraction Problems in Uniaxially Anisotropic Media", URSI Spring Meeting, Washington, D. C., 1962

C. J. Marcinkowski, "Plane Wave Scattering by a Linearly Varying Surface Impedance", Bull. Amer. Phys. Soc., Series II, Vol. 7, 31, Jan. 1962

A. A. Oliner and T. Tamir, "Radiation from Semi-Infinite Slot-Excited Plasma Sheath Configurations", Second Symposium on the Plasma Sheath, Boston, Mass. April 1962

J. Shmoys, "Diffraction by a Half-Plane with a Special Impedance Variation", Symposium on Electromagnetic Theory, Toronto, June 1959

T. Tamir and A. A. Oliner, "The Near Field Supported by a Source-Excited Plasma Slab", URSI-IRE, Washington, D. C., May 1961

T. Tamir and A. A. Oliner, "The Influence of Complex Waves on the Radiation Field of a Slot-Excited Plasma Layer", URSI-IRE, Austin, Texas, Nov. 1961

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